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ScienceDirect
Nuclear Physics B 967 (2021) 115427
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

Cubic vertices for $\mathcal{N} = 1$ supersymmetric massless higher spin fields in various dimensions

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Received 26 March 2021; accepted 30 April 2021
Available online 6 May 2021
Editor: Stephan Stieberger

Abstract

Using the BRST approach to higher spin field theories we develop a generic technique for constructing the cubic interaction vertices for $N = 1$ supersymmetric massless higher spin fields on four, six and ten dimensional flat backgrounds. Such an approach allows formulation of the equations for cubic vertices including bosonic and fermionic higher spin fields, and the problem of finding the vertices is reduced to finding the consistent solutions to these equations. As a realization of this procedure, we present the particular solutions for the vertices where the fields obey some off-shell constraints. It is shown that the supersymmetry imposes additional constraints on the vertices and singles out a particular subclass of the solutions. As a concrete application of the generic scheme, we consider supersymmetric Yang-Mills-like systems in four, six and ten dimensions where the higher spin fields transform under some internal symmetry group, as well as supergravity-like systems in the same dimensions.

1. Introduction

The problem of constructing interactions for higher spin fields has attracted much attention for many years. Interest in this problem is due to certain possibilities for the development of

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https://doi.org/10.1016/j.nuclphysb.2021.115427

new principles for constructing unified models of fundamental interactions, including quantum gravity, and phenomenological applications that are opening up in elementary particle physics and cosmology. Another fundamental principle that apparently should underlie the unified theory is related to supersymmetry. Therefore, the construction of a supersymmetric theory of higher-spin fields seems to be quite natural and relevant.

Until now supersymmetric higher spin theories have been studied mainly in three and four dimensional flat and AdS backgrounds (see [1–5] for earlier papers on the subject and [6–9] for recent reviews), whereas considerations in higher dimensions have been relatively rare [10]–[11]. Nevertheless, higher dimensional supersymmetric higher spin theories are interesting for several reasons. Firstly, they might prove to be helpful for further investigations of a connection between higher spin and string theories, since the latter also lives in higher dimensions. Secondly, higher dimensional higher spin theories can open some new possibilities for building lower dimensional supersymmetric models, as in supergravity, where the lower dimensional models can be obtained from relatively simpler higher dimensional ones via various kinds of compactifications and dimensional reductions.

Recently a particular class of free Lagrangians for $N=1$ supersymmetric massless higher spin fields was obtained [11], [12] for $D=3,4,6$ and 10 dimensional flat backgrounds. Although this construction bears a certain similarity with the supersymmetric open string field theory [13], it turns out that for massless higher spin fields one can build finite dimensional supermultiplets with the supersymmetry algebra being closed on-shell both in the bosonic (NS) and in the fermionic (R) sectors. To be more specific, the model obtained in [11] is a supersymmetrization of free Lagrangians for certain reducible representations of the Poincaré group and the specific structure of these representations is singled out by the requirement of $N=1$ supersymmetry. The fermionic sector (so-called "triplet" [14]–[15]) contains physical and auxiliary fields, which are totally symmetric with respect to their indices.[1] The physical field and a part of the auxiliary fields in the bosonic sector have indices of two types: n indices of one type and one index of another type. The indices of the first type are totally symmetric among each other, while there is no symmetry between them and the index of the second type. The other auxiliary fields are totally symmetric, i.e., they have indices only of the first type. This is the simplest example of so–called generalized triplets [17] which describes reducible representations of the Poincaré group for fields with mixed symmetries. In both the fermionic and bosonic sectors the auxiliary fields are eliminated via their own equations of motion and/or after the complete gauge fixing so the system describes on-shell only physical polarizations. To summarize, in the fermionic sector the physical fields are described by the rank n spin tensor which contains the fields with spins $n+1/2, n-1/2, ..., 1/2$ The bosonic sector contains the physical fields described by Young tableaux with two rows. These Young tableaux are of the type $(n,1)$ and $(n+1,0)$.

As mentioned above, the triplets are reducible representations of the Poincaré group and unlike the so-called Fronsdal fields [18]–[19], each triplet contains more than one physical field. These fields correspond to single, double, etc. traces of the tensor/spin-tensor field of rank n. On the other hand, the question of whether the corresponding supermultiplets are reducible or not in the sense of representations of SUSY algebra has different answers depending on the space-time dimensions, as they can be either reducible or irreducible. This can be easily seen for the example of the lower spin fields. The lowest spin case which corresponds to $n=0$, describes the $N=1$ supersymmetric Maxwell theory in $D=4,6$ and 10 and is in some sense degenerate. The

[1] A frame-like formulation of triplets was obtained in [16].

next simplest case, with $n = 1$ corresponds to linearized $N = 1$ supergravity theories. The Lagrangians of [11] describe irreducible supermultiplets for $D = 10$ and reducible supermultiplets for $D = 4$ and $D = 6$. Proceeding further, one can show that the situation for the higher spin fields is the same as for the case of linearized supergravity multiplets.

It is a natural next step to study a possibility for cubic interactions for the free systems described above. The problem of construction and of the further study of cubic vertices for massless and massive higher spin fields has been attracting considerable interest [20–31] (see also [32–34] for earlier work and [35–39] for supersymmetric and non-supersymmetric cubic interactions on AdS backgrounds in the frame–like approach). Although these studies were mainly devoted to non-supersymmetric theories, several interesting results appeared recently for supersymmetric cubic vertices in four dimensions. In particular, in [40]– [41] cubic vertices for $N = 1$ superfields and for extended N were obtained in the light-cone approach. In the papers [42–48] the interactions between conserved higher spin supercurrents and chiral superfields, as well as interactions between supersymmetric sigma-models and higher spin superfields on flat and AdS backgrounds were constructed. The cubic interactions for supersymmetric systems were also recently constructed in [49]– [50].

In the present paper we extend, at least partially, the results of [11] by including cubic interaction vertices into consideration. By "partially" we mean the following: for the purpose of simplifying the computations, we shall partially gauge fix the free Lagrangians, so the fields contain only physical transverse components. As the second step we perform nonlinear deformations of these Lagrangians by including cubic interaction vertices. It turns out however, that because of this gauge fixing, a further requirement of the invariance under $N = 1$ supersymmetry transformations puts the fields completely on shell. These completely on-shell vertices can be promoted back to the off-shell ones by including all auxiliary fields into the free and interacting Lagrangians i.e., by considering the system given in [11] without gauge fixing and then repeating the procedure described above.[2] The "physical" part of these vertices, which does not contain any auxiliary fields will coincide with the ones obtained in the present paper. Here we shall present the defining equations for these vertices and leave the detailed analysis for a separate publication.

The paper is organized as follows:

In section 2 we give a brief description of the free supersymmetric systems for which we are going to build cubic interactions. To this end, we use the BRST approach.[3] which yields the free Lagrangians given in [11] with no off-shell constraints neither on the fields under consideration nor on the parameter of gauge transformations. We then gauge fix the Lagrangians, so that they contain only physical components, and the fields and parameters of gauge transformations obey certain off–shell constraints[4]

Section 3, where we describe the cubic interactions, contains two subsections. In subsection 3.1 we collect the expressions for the vertices for three bosonic higher spin fields both in an unconstrained and in a gauge fixed form [26], [22]. These vertices correspond to the purely bosonic

[2] Usually in the context of supersymmetric theories the words "on-shell" and "off-shell" indicate whether the supersymmetry algebra is closed after taking into account the equations of motion or not (see e.g. [51]). The $N = 1$ systems constructed in [11] are formulated in terms of component fields, i.e., they are "on-shell". Here "completely on-shell" means that we use the field equations in order to have the cubic vertices transformed into each other under the supersymmetry transformations.

[3] Originally used for a description of totally symmetric massless and massive reducible representations of the Poincaré group [52]–[53], this approach was then generalized for description of Poincaré and AdS_D groups, see e.g. [54–63].

[4] This is called a constrained formulation in [64] whereas the formulation where no off-shell constraints are imposed is called unconstrained one [65].

part of cubic interactions of the systems under consideration. The equations that determine cubic vertices for two fermionic and one bosonic higher spin fields are given in subsection 3.2.

In section 4 we describe the higher spin generalization of the $N = 1$ super Yang-Mills theory. First we derive the corresponding cubic vertices and the Lagrangians, which describe the cubic interactions between reducible massless representations of the Poincaré group. These vertices are a covariant form of the analogous vertices obtained in the light-front formalism in [22]. Then we present the $N = 1$ supersymmetry transformation for these systems. As mentioned above, the requirement of the supersymmetry puts the fields on shell, because of the form of the gauge fixing. A somewhat degenerate case of $N = 1$ super Yang-Mills theory which illustrates how the whole system can be promoted to an off-shell description is given separately.

An analogous consideration for the higher spin generalization of the four dimensional $N = 1$ supergravity is given in section 5.

The last section contains our conclusions and the summary of results.

Some lengthy equations and useful identities for gamma matrices and for linearized gravity are collected in the appendices.

2. Free Lagrangians

In this section we shall briefly describe free Lagrangians for bosonic and fermionic massless higher spin fields, whose cubic deformations and supersymmetrizations we are going to consider.

As we mentioned in the introduction, in the fermionic F sector we have n totally symmetric spin-tensor fields, both physical and auxiliary. In the bosonic sector B the physical field contains n indices which are symmetric among each other and one index which has no symmetry with the other ones. The auxiliary fields in the bosonic sector are either totally symmetric or have mixed symmetry.

In order to derive the corresponding Lagrangians, we introduce commuting oscillators $\alpha_m^{\mu,\pm}$, anticommuting ghosts $c_m^{\pm}$, c_0 and antighosts $b_m^{\pm}$, b_0, where $m = 1, 2$ in the B sector and $m = 1$ in the F sector. These oscillators obey the following (anti)commutation relations

$$[\alpha_m^{\mu}, \alpha_n^{\nu,+}] = \eta^{\mu\nu}\delta_{mn}, \quad \{c_m^+, b_n\} = \{c_m, b_n^+\} = \delta_{mn}\,, \quad \{c_0, b_0\} = 1. \tag{2.1}$$

The ghost number of $c_m^{\pm}$ and c_0 is $+1$, the ghost number of $b_m^{\pm}$ and b_0 is consequently -1 and the ghost number of $\alpha_m^{\mu,\pm}$ is zero.

The Fock vacua in the B and in F sectors are defined as, respectively

$$\alpha_m^{\mu}|0_B\rangle = c_m|0_B\rangle = b_m|0_B\rangle = b_0|0_B\rangle = 0, \qquad m = 1, 2. \tag{2.2}$$

$$\alpha_1^{\mu}|0_F\rangle = c_1|0_F\rangle = b_1|0_F\rangle = b_0|0_F\rangle = 0. \tag{2.3}$$

Higher spin functionals either in the B or F sector, are expanded in terms of the creation operators and the components of this expansion are higher spin fields (physical and auxiliary).

Let us now introduce differential operators. In the B sector we have

$$l_0 = p \cdot p, \qquad l_m = p \cdot \alpha_m, \qquad l_m^+ = p \cdot \alpha_m^+, \tag{2.4}$$

where $p_\mu = -i\partial_\mu$ when acting to the right. The symbol 'dot' means $A \cdot B = \eta_{\mu\nu}A^\mu B^\nu$ and ∂A denotes a symmetrized derivative. For example if A is a vector, then $\partial A \equiv \partial_\mu A_\nu + \partial_\nu A_\mu$. Obviously l_0 is the d'Alembertian, l_m being divergence operators with respect to the indices contracted with $\alpha_m^{\mu,+}$ oscillators and l_m^+ being derivatives symmetrized with the indices contracted with $\alpha_m^{\mu,+}$ oscillators. Alternatively, one can work in a momentum representation, without realizing p_μ as a differential operator.

In the F sector we have

$$l_1 = p \cdot \alpha_1, \qquad l_1^+ = p \cdot \alpha_1^+, \quad g_0 = p \cdot \gamma, \tag{2.5}$$

where γ_μ are gamma-matrices and the operator g_0 being the Dirac operator.

Having defined all necessary operators we can write a free Lagrangian for bosonic fields as

$$\mathcal{L}_{B,\text{free}} = \int dc_0 \langle \Phi_B | Q_B | \Phi_B \rangle, \tag{2.6}$$

with the corresponding nilpotent BRST charge

$$Q_B \;=\; c_0 \, l_0 \;+\; \tilde{Q}_B \;-\; M_B \, b_0 \;, \tag{2.7}$$

where

$$\tilde{Q}_B \;=\; \sum_{m=1,2} (c_m^+ l_m + c_m l_m^+), \qquad M_B \;=\; \sum_{m=1,2} c_m^+ c_m. \tag{2.8}$$

The Lagrangian (2.6) is invariant under the gauge transformations

$$\delta |\Phi_B\rangle = Q_B |\Lambda_B\rangle \tag{2.9}$$

due to the nilpotency of the BRST charge (2.7) in any space-time dimension D. The ghost number of the field $|\Phi_B\rangle$ is fixed to be zero and consequently the ghost number of the parameter of gauge transformations $|\Lambda_B\rangle$ is equal to -1.

Further, in order to establish $N = 1$ supersymmetry, one requires that the component of the higher spin functional $|\Phi_B\rangle$, which does not depend on ghosts/antighosts and describes the physical field, depends on the oscillator $\alpha_2^{+\nu}$ only linearly. The explicit form of $|\Phi_B\rangle$ and $|\Lambda_B\rangle$ can be completely fixed and is given in [11].

In the following we shall work with the gauge fixed form of the Lagrangian (2.6) by imposing off-shell conditions $|\Phi_B\rangle$

$$l_m |\Phi_B\rangle = 0, \quad m = 1, 2. \tag{2.10}$$

As a result the higher spin functional contains only a physical field

$$|\Phi_B\rangle \equiv |\phi\rangle = \frac{1}{n!}\, \phi_{\mu_1\mu_2\ldots\mu_n,\nu}(x)\, \alpha_2^{\nu+} \alpha_1^{\mu_1+}\, \alpha_1^{\mu_2+} \ldots \alpha_1^{\mu_n+}\, |0_B\rangle, \tag{2.11}$$

all ghost dependence being gauged away. The physical field obeys off-shell transversality conditions, i.e., we are essentially dealing with only physical components.

The Lagrangian (2.6) and the gauge fixing conditions (2.10) are still invariant under the gauge transformations (2.9) with the parameter of gauge transformations

$$\begin{aligned} |\Lambda_B\rangle &= b_1^+ |\lambda\rangle + b_2^+ |\rho\rangle = \\ &= \frac{i b_1^+}{(n-1)!} \lambda_{\nu,\mu_1\mu_2\ldots\mu_{n-1}}(x)\, \alpha_2^{\nu+}\, \alpha_1^{\mu_1+}\, \alpha_1^{\mu_2+} \ldots \alpha_1^{\mu_{n-1}+}\, |0_B\rangle + \\ &+ \frac{i b_2^+}{n!} \rho_{\mu_1\mu_2\ldots\mu_n}(x)\, \alpha_1^{\mu_1+}\, \alpha_1^{\mu_2+} \ldots \alpha_1^{\mu_n+}\, |0_B\rangle \end{aligned} \tag{2.12}$$

being restricted as

$$l_0|\Lambda_B\rangle = l_m|\Lambda_B\rangle = 0, \quad m = 1, 2 \tag{2.13}$$

The free Lagrangian for the fermionic triplet is

$$\mathcal{L}_{F,\text{free}} = \frac{1}{\sqrt{2}}\, {}_a\langle\Phi_{F,1}|(g_0)^a{}_b|\Phi_{F,1}\rangle^b + {}_a\langle\Phi_{F,2}|\tilde{Q}_F|\Phi_{F,1}\rangle^a + \\ + {}_a\langle\Phi_{F,1}|\tilde{Q}_F|\Phi_{F,2}\rangle^a + \sqrt{2}\, {}_a\langle\Phi_{F,2}|M_F(g_0)^a{}_b|\Phi_{F,2}\rangle^b, \tag{2.14}$$

where

$$\tilde{Q}_F = c_1^+ l_1 + c_1 l_1^+, \qquad M_F = c_1^+ c_1. \tag{2.15}$$

The Lagrangian contains two fields, each of them being a series of expansion in terms of the creation operators $\alpha_1^{\nu,+}$, c_1^+, and b_1^+. The field $|\Phi_{F,1}\rangle^b$ contains physical and auxiliary fields, while the field $|\Phi_{F,2}\rangle^a$ is purely auxiliary. The Lagrangian (2.14) is invariant under the gauge transformations

$$\begin{aligned} \delta\,|\Phi_{F,1}\rangle^a &= \tilde{Q}_F|\Lambda_{F,1}\rangle^a + \sqrt{2}\,M_F\,(g_0)^a{}_b\,|\Lambda_{F,2}\rangle^b\,, \\ \delta\,|\Phi_{F,2}\rangle^a &= -\frac{1}{\sqrt{2}}(g_0)^a{}_b\,|\Lambda_{F,1}\rangle^b - \tilde{Q}_F\,|\Lambda_{F,2}\rangle^a \end{aligned} \tag{2.16}$$

with unconstrained parameters $|\Lambda_{F,1}\rangle^a$ and $|\Lambda_{F,2}\rangle^a$. The fields $|\Phi_{F,1}\rangle^a$ and $|\Phi_{F,2}\rangle^a$ have ghost numbers 0 and -1 respectively. The parameters of gauge transformations $|\Lambda_{F,1}\rangle^a$ and $|\Lambda_{F,2}\rangle^a$ have ghost numbers -1 and -2 respectively. Similarly to the bosonic sector, one can consider only a physical field

$$|\Phi_F\rangle^a \equiv |\Psi\rangle^a = \frac{1}{n!}\,\Psi^a_{\mu_1\mu_2\dots\mu_n}(x)\,\alpha_1^{\mu_1+}\,\alpha_1^{\mu_2+}\dots\alpha_1^{\mu_n+}\,|0_F\rangle. \tag{2.17}$$

In this gauge the field $|\Phi_F\rangle^a$ does not depend on ghost/antighost variables and obeys the off-shell transversality condition

$$l_1|\Phi_F\rangle^a = 0. \tag{2.18}$$

The free Lagrangian in the fermionic sector is therefore

$$\mathcal{L}_{F.free} = {}_a\langle\Phi_F|(g_0)^a{}_b|\Phi_f\rangle^b. \tag{2.19}$$

The off-shell constraints (2.18) and the Lagrangian (2.19) are invariant under the transformations

$$\delta\,|\Phi_F\rangle^a = \tilde{Q}_F|\Lambda_F\rangle^a \tag{2.20}$$

provided the parameter of gauge transformations

$$|\Lambda_F\rangle^a = b_1^+|\tilde{\lambda}\rangle^a = \frac{ib_1^+}{(n-1)!}\,\tilde{\lambda}^a_{\mu_1\mu_2\dots\mu_{n-1}}(x)\,\alpha_1^{+\mu_1}\,\alpha_1^{+\mu_2}\dots\alpha_1^{+\mu_{n-1}}\,|0_F\rangle\,, \tag{2.21}$$

is constrained as

$$(g_0)^a{}_b|\Lambda_F\rangle^b = l_1|\Lambda_F\rangle^a = 0. \tag{2.22}$$

Finally, let us note that all representations of the Poincaré group that we discussed in this section are reducible since no (gamma)tracelessness condition has been imposed at any stage.

3. Cubic interactions

3.1. Three bosons

Below we shall follow the approach of [21] for the construction of off-shell cubic interaction vertices between three bosonic higher spin fields. This method is a modification of the construction developed in open string field theory [66] – [67] to the case of higher spin fields and it can be applied for either massless or massive fields, both on flat and $(A)dS_D$ backgrounds.

Let us take three copies of the Fock spaces introduced in section 2. All oscillators get an extra index $i = 1, 2, 3$ and the nonzero commutation relations are only between oscillators, which belong to the same Fock space

$$[\alpha^{(i)}_{\mu,m}, \alpha^{(j),+}_{\nu,n}] = \delta^{ij}\delta_{mn}\eta_{\mu\nu}, \tag{3.1}$$

$$\{c^{(i),+}_m, b^{(j)}_n\} = \{c^{(i)}_m, b^{(j),+}_n\} = \{c^{(i)}_{0,m}, b^{(j)}_{0,n}\} = \delta^{ij}\delta_{mn}\,. \tag{3.2}$$

The operator $p^{(i)}_\mu$ corresponds to the momentum in the i-th Fock space. In a coordinate representation the expression $p^{(i)}_\mu = -i\partial^{(i)}_\mu$ is a derivative acting on the fields in i-th Fock space. The momentum operators obey the constraint

$$p^{(1)}_\mu + p^{(2)}_\mu + p^{(3)}_\mu = 0. \tag{3.3}$$

Finally, we also allow the higher spin functionals to carry some internal symmetry indices denoted as A, B, C.

Next, we consider the Lagrangian

$$\begin{aligned}\mathcal{L}_{B,\text{int}} = &\sum_{i=1}^{3}\int dc^{(i)A}_0 \langle\Phi^{(i)}_B|Q^{(i)}_B|\Phi^{(i)}_B\rangle_A + \\ &+ g\left(\int dc^{(1)}_0 dc^{(2)}_0 dc^{(3)A}_0 \langle\Phi^{(1)}_B|^B\langle\Phi^{(2)}_B|^C\langle\Phi^{(3)}_B||V\rangle_{ABC} + h.c.\right)\end{aligned} \tag{3.4}$$

and modified gauge transformations

$$\begin{aligned}\delta|\Phi^{(1)}_B\rangle_A = &\, Q^{(1)}_B|\Lambda^{(1)}_B\rangle_A - \\ &- g\int dc^{(2)}_0 dc^{(3)}_0 \left(({}^B\langle\Phi^{(2)}_B|^C\langle\Lambda^{(3)}_B| + {}^C\langle\Phi^{(3)}_B|^B\langle\Lambda^{(2)}_B|)|V\rangle_{ABC}\right)\end{aligned} \tag{3.5}$$

where g is a coupling constant. The transformations for the fields $|\Phi^{(2)}_B\rangle_A$ and $|\Phi^{(3)}_B\rangle_A$ are obtained from (3.5) via cyclic permutations.

Below, we will consider two types of vertices. In one type of vertices, to which we refer as "gravity-like" vertices, the internal indices are absent. The other type of vertices, referred to as "Yang-Mills-like", has the form

$$|V\rangle_{ABC} = f_{ABC}|V\rangle \tag{3.6}$$

for some totally antisymmetric structure constants f_{ABC}.

In both cases the invariance of the Lagrangian (3.4) under the transformations (3.5) in the zeroth order in g is maintained due to the nilpotency of the BRST charges in each Fock space

$$(Q^{(1)}_B)^2 = (Q^{(2)}_B)^2 = (Q^{(3)}_B)^2 = 0. \tag{3.7}$$

The invariance at the first order in g implies that the cubic vertex is BRST invariant

$$(Q_B^{(1)} + Q_B^{(2)} + Q_B^{(3)})|V\rangle = 0 \tag{3.8}$$

The condition (3.8) also guarantees that the group structure of the gauge transformations is preserved at the first order in g.

The cubic vertex has a general structure

$$|V\rangle = V\ c_0^{(1)}c_0^{(2)}c_0^{(3)}\ |0_B^{(1)}\rangle \otimes |0_B^{(2)}\rangle \otimes |0_B^{(3)}\rangle \tag{3.9}$$

where an unknown function V can depend on $p_\mu^{(i)}, \alpha_\mu^{(i),+}, c^{(i),+}, b^{(i),+}, b_0^{(i)+}$. Apart from the condition of BRST invariance (3.8), the function V is required to be Lorentz invariant and to have zero ghost number.

It can be verified by direct computations [26], [29] that the following expressions are BRST invariant and therefore any function of them is a solution of (3.8)

$$\mathcal{K}_m^{(i)} = (p^{(i+1)} - p^{(i+2)}) \cdot \alpha_m^{(i),+} + (b_0^{(i+1)} - b_0^{(i+2)})\, c_m^{(i),+}, \tag{3.10}$$

$$\mathcal{O}_{mn}^{(i,i)} = \alpha_m^{(i),+} \cdot \alpha_n^{(i),+} + c_m^{(i),+} b_n^{(i),+} + c_n^{(i),+} b_m^{(i),+}, \tag{3.11}$$

$$\mathcal{Z}_{mnp} = \mathcal{Q}_{mn}^{(1,2)}\mathcal{K}_p^{(3)} + \mathcal{Q}_{np}^{(2,3)}\mathcal{K}_m^{(1)} + \mathcal{Q}_{pm}^{(3,1)}\mathcal{K}_n^{(2)}, \tag{3.12}$$

where[5]

$$\mathcal{Q}_{mn}^{(i,i+1)} = \alpha_m^{(i),+} \cdot \alpha_n^{(i+1),+} + \frac{1}{2} b_m^{(i),+} c_n^{(i+1),+} + \frac{1}{2} b_n^{(i+1),+} c_m^{(i),+}. \tag{3.13}$$

After the gauge fixing described in section 2 the Lagrangian (3.4) simplifies to

$$\mathcal{L}_{\rm int} = \sum_{i=1,2,3} {}^{A}\langle \phi^{(i)}| l_0^{(i)} |\phi_B^{(3)}\rangle_A + \left({}^{C}\langle \phi^{(1)}|\ {}^{A}\langle \phi^{(2)}|\ {}^{B}\langle \phi^{(3)}|| V\rangle_{ABC} + h.c \right) \tag{3.14}$$

and describes cubic interactions between three massless higher spin fields without their auxiliary components. As we shall see below, further requirement of $N = 1$ supersymmetry will single out some particular subclasses of the cubic vertices.

3.2. *Two fermions and one boson*

Cubic interactions between two fermionic and one bosonic higher spin fields can be treated in the BRST approach in a similar way. However, there is one important difference, which makes the present case technically more complicated. This difference shows up already at the level of the free Lagrangians: because of the absence of the ghost c_0 (see section 2 and [11], [17] for details) the free Lagrangian for the fermionic triplets contains two different operators: the Dirac operator g_0 and operator (2.15), instead of only one BRST charge (2.7) present in the Lagrangians for free bosonic triplets.

Making a cubic deformation of the free Lagrangian we get

$$\mathcal{L}_{\rm int} = \sum_{i=1,2} (\frac{1}{\sqrt{2}}\ {}_a^A\langle \Phi_{F,1}^{(i)}| (g_0^{(i)})^a{}_b |\Phi_{F,1}^{(i)}\rangle_A^b + {}_a^A\langle \Phi_{F,2}^{(i)}| \tilde{Q}_F^{(i)} |\Phi_{F,1}^{(i)}\rangle_A^a + \tag{3.15}$$

[5] Some solutions of (3.8) can be obtained from the free Lagrangian by the field redefinitions. Such vertices have the form $|V\rangle = \sum_{i=1,2,3} Q_B^{(i)}|W^{(i)}\rangle$. Trivial vertices generically contain powers of $l_0^{(i)}$ and $l_m^{(i)}$ (defined in (2.4)) and therefore can be eliminated [21].

$$
\begin{aligned}
&+ {}_a^A\langle \Phi^{(i)}_{F,1}|\tilde{Q}^{(i)}_F|\Phi^{(i)}_{F,2}\rangle^a_A + \sqrt{2}\, {}_a^A\langle \Phi^{(i)}_{F,2}|M^{(i)}_F (g^{(i)}_0)^a{}_b|\Phi^{(i)}_{F,2}\rangle^b_A) \\
&+ \int dc^{(3)}_0 {}^A\langle \Phi^{(3)}_B|Q^{(3)}_B|\Phi^{(3)}_B\rangle_A + \\
&+ g \sum_{m,n=1,2} \int dc^{(3)}_0 \left({}^C\langle \Phi^{(3)}_B|\, {}_a^A\langle \Phi^{(1)}_{F,m}|\, {}_b^B\langle \Phi^{(2)}_{F,n}||\mathcal{V}_{mn}\rangle^{ab}_{ABC} + h.c \right).
\end{aligned}
$$

The structure of the cubic vertex

$$
|\mathcal{V}\rangle^{ab}_{ABC} = \mathcal{V}^{ab}_{ABC}\ |0^{(1)}_F\rangle \otimes |0^{(2)}_F\rangle \otimes c^{(3)}_0 |0^{(3)}_B\rangle, \tag{3.16}
$$

and the fact that the Lagrangian and the higher spin functionals have the ghost number zero, implies that the unknown function $\mathcal{V}^{ab}_{ABC}$ has ghost number zero as well. The requirement of the invariance of the Lagrangian (3.15) under the nonlinear gauge transformations

$$
\begin{aligned}
\delta|\Phi^{(3)}_B\rangle_C &= Q^{(3)}_B|\Lambda^{(3)}_B\rangle_C + \\
&\quad + g \sum_{m,n=1,2} ({}_a^A\langle \Phi^{(1)}_{F,m}|\, {}_b^B\langle \Lambda_{F,n}{}^{(2)}||\mathcal{W}^{1,2}_{3,mn}\rangle^{ab}_{ABC} + \\
&\quad + {}_a^A\langle \Phi^{(2)}_{F,m}|\, {}_b^B\langle \Lambda^{(1)}_{F,n}|\, |\mathcal{W}^{2,1}_{3,mn}\rangle^{ab}_{ABC})
\end{aligned} \tag{3.17}
$$

$$
\begin{aligned}
\delta|\Phi^{(i)}_{F,1}\rangle^a_C &= \tilde{Q}^{(i)}_F|\Lambda_{F,1}{}^{(i)}\rangle^a_C + \sqrt{2}\, M^{(i)}_F\, (g^{(i)}_0)^a{}_b\, |\Lambda^{(i)}_{F,2}\rangle^b_C \\
&\quad + g \sum_{m=1,2} \int dc^{(3)}_0 ({}_b^A\langle \Phi^{(3-i)}_{F,m}|\, {}^B\langle \Lambda^{(3)}_B||\mathcal{W}^{3-i,3}_{i,1m}\rangle^{ab}_{ABC} + \\
&\quad + {}^A\langle \Phi^{(3)}_B|\, {}_b^B\langle \Lambda^{(3-i)}_{F,m}||\mathcal{W}^{3,3-i}_{i,1m}\rangle^{ab}_{ABC})
\end{aligned} \tag{3.18}
$$

$$
\begin{aligned}
\delta|\Phi^{(i)}_{F,2}\rangle^a_C &= -\frac{1}{\sqrt{2}}(g^{(i)}_0)^a{}_b\, |\Lambda^{(i)}_{F,1}\rangle^b_C \ -\ \tilde{Q}^{(i)}_F\ |\Lambda^{(i)}_{F,2}\rangle^a_C \\
&\quad + g \sum_{m=1,2} \int dc^{(3)}_0 ({}_b^A\langle \Phi^{(3-i)}_{F,m}|\, {}^B\langle \Lambda^{(3)}_B||\mathcal{W}^{3-i,3}_{i,2m}\rangle^{ab}_{ABC} + \\
&\quad + {}^A\langle \Phi^{(3)}_B|\, {}_b^B\langle \Lambda^{(3-i)}_{F,m}||\mathcal{W}^{3,3-i}_{i,2m}\rangle^{ab}_{ABC})
\end{aligned} \tag{3.19}
$$

leads to equations for unknown vertices $|\mathcal{V}_{mn}\rangle^{ab}_{ABC}$ and $|\mathcal{W}^{ij}_{k,mn}\rangle^{ab}_{ABC}$ which are given in (B.1)–(B.12).

One can however consider a simpler problem, where the higher spin functionals are gauge fixed, as was discussed in section 2. At the cubic level this simply means considering only physical (ghost independent) components in (3.15) and integrating out of the ghost zero mode. Then the corresponding cubic Lagrangian has the form

$$
\begin{aligned}
\mathcal{L}_{\text{int}} &= \sum_{i=1}^{2} {}_a^A\langle \Psi^{(i)}|(g^{(i)}_0)^a{}_b|\Psi^{(a)}\rangle^b_A + {}^A\langle \phi^{(3)}|l^{(3)}_0|\phi^{(3)}\rangle_A + \\
&\quad + g\left({}^C\langle \phi^{(3)}|\, {}_a^A\langle \Psi^{(1)}|\, {}_b^B\langle \Psi^{(2)}||\mathcal{V}\rangle^{ab}_{ABC} + h.c \right).
\end{aligned} \tag{3.20}
$$

The invariance under nonlinear gauge transformations

$$
\begin{aligned}
\delta|\phi^{(3)}\rangle_C &= \tilde{Q}^{(3)}_B|\Lambda^{(3)}_B\rangle_C + \\
&\quad + g({}_a^A\langle \Psi^{(1)}|\, {}_b^B\langle \Lambda_F{}^{(2)}||\mathcal{W}^{1,2}_3\rangle^{ab}_{ABC} + {}_a^A\langle \Psi^{(2)}|\, {}_b^B\langle \Lambda^{(1)}_F|\, |\mathcal{W}^{2,1}_3\rangle^{ab}_{ABC}),
\end{aligned} \tag{3.21}
$$

$$\delta|\Psi^{(1)}\rangle^a_C = \tilde{Q}^{(1)}_F |\Lambda_F{}^{(1)}\rangle^a_C + \\ + g({}^A_b\langle\Psi^{(2)}|\ {}^B\langle\Lambda^{(3)}_B||\mathcal{W}^{2,3}_1\rangle^{ab}_{ABC} + {}^A\langle\phi^{(3)}|\ {}^B_b\langle\Lambda^{(2)}_F||\mathcal{W}^{3,2}_1\rangle^{ab}_{ABC}), \tag{3.22}$$

$$\delta|\Psi^{(2)}\rangle^a_C = \tilde{Q}^{(2)}_F |\Lambda_F{}^{(2)}\rangle^a_C + \\ + g({}^A\langle\phi^{(3)}|\ {}^B_b\langle\Lambda^{(1)}_F||\mathcal{W}^{3,1}_2\rangle^{ab}_{ABC} + {}^A_b\langle\Psi^{(1)}|\ {}^B\langle\Lambda^{(3)}_B||\mathcal{W}^{1,3}_2\rangle^{ab}_{ABC}), \tag{3.23}$$

implies the following conditions on the vertices

$${}^B\langle\Lambda^{(3)}_B|^C_a\langle\Psi^{(1)}|^A_c\langle\Psi^{(2)}|\Big((g^{(1)}_0)^a{}_b|\mathcal{W}^{2,3}_1\rangle^{bc}_{ABC} - (g^{(2)}_0)^c{}_b|\mathcal{W}^{1,3}_2\rangle^{ba}_{CBA} + \tilde{Q}^{(3)}_B|\mathcal{V}\rangle^{ac}_{CAB}\Big) = 0 \tag{3.24}$$

$${}^C\langle\phi^{(3)}|^A_a\langle\Psi^{(1)}|^B_c\langle\Lambda^{(2)}_F|\Big((g^{(1)}_0)^a{}_b|\mathcal{W}^{3,2}_1\rangle^{bc}_{CBA} + l^{(3)}_0|\mathcal{W}^{1,2}_3\rangle^{ac}_{ABC} + \tilde{Q}^{(2)}_F|\mathcal{V}\rangle^{ac}_{ABC}\Big) = 0 \tag{3.25}$$

$${}^C\langle\phi^{(3)}|^B_c\langle\Lambda^{(1)}_F|^A_a\langle\Psi^{(2)}|\Big((g^{(2)}_0)^a{}_b|\mathcal{W}^{3,1}_2\rangle^{bc}_{CBA} + l^{(3)}_0|\mathcal{W}^{2,1}_3\rangle^{ac}_{ABC} - \tilde{Q}^{(1)}_F|\mathcal{V}\rangle^{ca}_{BAC}\Big) = 0 \tag{3.26}$$

where $|\Psi^{(i)}\rangle$, $|\phi^{(3)}\rangle$, $|\Lambda^{(i)}_B\rangle$, and $|\Lambda^{(3)}_F\rangle$ are constrained as described in section 2.

Furthermore, for the preservation of the group structure of the gauge transformations up to the first order in the coupling constant g there must exist some functions $|\mathcal{X}_i\rangle$ such that

$${}^A_b\langle\Lambda^{(2)}_F|^B\langle\Lambda^{(3)}_B|\Big(\tilde{Q}^{(2)}_F|\mathcal{W}^{2,3}_1\rangle^{ab}_{ABC} + \tilde{Q}^{(3)}_B|\mathcal{W}^{3,2}_1\rangle^{ab}_{BAC} - \tilde{Q}^{(1)}_F|\mathcal{X}_1\rangle^{ab}_{ABC}\Big) = 0 \tag{3.27}$$

$${}^B\langle\Lambda^{(3)}_B|^A_b\langle\Lambda^{(1)}_F|\Big(\tilde{Q}^{(1)}_F|\mathcal{W}^{1,3}_2\rangle^{ab}_{ABC} + \tilde{Q}^{(3)}_B|\mathcal{W}^{3,1}_2\rangle^{ab}_{BAC} - \tilde{Q}^{(2)}_F|\mathcal{X}_2\rangle^{ab}_{ABC}\Big) = 0 \tag{3.28}$$

$${}^A_a\langle\Lambda^{(1)}_F|^B_b\langle\Lambda^{(2)}_F|\Big(\tilde{Q}^{(1)}_F|\mathcal{W}^{1,2}_3\rangle^{ab}_{ABC} - \tilde{Q}^{(2)}_F|\mathcal{W}^{2,1}_3\rangle^{ba}_{BAC} - \tilde{Q}^{(3)}_B|\mathcal{X}_3\rangle^{ab}_{ABC}\Big) = 0 \tag{3.29}$$

Since both the Lagrangian and the higher spin functionals have ghost number zero, it follows that the vertex $|\mathcal{V}\rangle$ has the ghost number 0 and the $|\mathcal{W}\rangle$ and $|\mathcal{X}\rangle$– vertices have ghost number $+1$.

Let us note, that the Lagrangian (3.20) is symmetric under the exchange of $|\Psi^{(1)}\rangle$ and $|\Psi^{(2)}\rangle$, provided the vertex obeys the symmetry

$$1 \leftrightarrow 2, \qquad |\mathcal{V}\rangle^{ab}_{ABC} \to -|\mathcal{V}\rangle^{ba}_{BAC} \tag{3.30}$$

The gauge transformation rules (3.21)-(3.23) are symmetric under the exchange of labels 1 and 2 as well. Similarly, the transformation (3.30) leaves the equation (3.24) invariant, and takes the equation (3.25) to (3.26) and vice versa.

To summarize, the vertex $|\mathcal{V}\rangle^{ab}$ describes the Lagrangian cubic interactions and the $|\mathcal{W}\rangle^{ab}$–vertices describe nonlinear deformations of the linear gauge transformations. The defining equations are (3.24)–(3.26), whereas the equations (3.27)–(3.29) are in a sense the consistency conditions for the vertices.

4. Super Yang-Mills-like systems

4.1. Vertices

Let us consider a vertex

$$(\mathcal{V})^{ab}_{ABC} = f_{ABC}(\gamma\cdot\alpha^+_2)^{ab}\mathcal{F}(\mathcal{K}^{(i)}_1, \mathcal{Z}_{111}) \tag{4.1}$$

for cubic interactions between two fermions and one boson. The function $\mathcal{F}$ is an arbitrary function of $\mathcal{Z}_{111}$ and $\mathcal{K}_1^{(i)}$, as defined in (3.10) and (3.12). We can solve explicitly equations (3.24)-(3.29) for any $\mathcal{F}$, with the solutions given in appendix C.

To simplify the following and aid in establishing supersymmetry we impose a cyclic symmetry on the vertex, by choosing $\mathcal{F}$ such that

$$\frac{\partial \mathcal{F}}{\partial \mathcal{K}_1^{(1)}} = \frac{\partial \mathcal{F}}{\partial \mathcal{K}_1^{(2)}} = \frac{\partial \mathcal{F}}{\partial \mathcal{K}_1^{(3)}} \equiv \frac{\partial \mathcal{F}}{\partial \mathcal{K}_1} \tag{4.2}$$

In order to consider $N = 1$ supersymmetry we shall choose the following cubic vertex for three bosonic higher spin fields

$${}^{C}\langle \phi^{(1)}| \, {}^{A}\langle \phi^{(2)}| \, {}^{B}\langle \phi^{(3)}| \mathcal{Z}_{222} \mathcal{F}(\mathcal{K}_1, \mathcal{Z}_{111}) |0_B^{(1)}\rangle \otimes |0_B^{(2)}\rangle \otimes |0_B^{(3)}\rangle f_{ABC} \tag{4.3}$$

The interaction between two fermions and one boson is described by the cubic vertex (4.1). However, for the purpose of finding supersymmetry transformations, we take three Fock spaces in the fermionic sector as well and consider the interactions between two fermions and one boson as

$${}^{C}\langle \phi^{(3)}| \, {}^{A}_{a}\langle \Psi^{(1)}| \, {}^{B}_{b}\langle \Psi^{(2)}| (\gamma \cdot \alpha_2^{(3),+})^{ab} \mathcal{F}(\mathcal{K}_1, \mathcal{Z}_{111}) |0_F^{(1)}\rangle \otimes |0_F^{(2)}\rangle \otimes |0_B^{(3)}\rangle f_{ABC} + \text{ cyclic} \tag{4.4}$$

Given the symmetry of exchanging the Fock space labels (3.30) in the definition of the vertices, the function $\mathcal{F}$ has to be even, in the sense that

$$\mathcal{F}(-\mathcal{K}_1, -\mathcal{Z}_{111}) = \mathcal{F}(\mathcal{K}_1, \mathcal{Z}_{111}) \tag{4.5}$$

This implies that the total number of α_1^+ oscillators in $\mathcal{F}$ is even, and the total number of oscillators in the vertices is odd.

Naturally, in order to establish supersymmetry for the nonlinear systems under consideration, one starts with the transformations that connect the free Lagrangians for fermionic and bosonic (generalized) triplets [11]

$$\delta |\phi^{(i)}\rangle_A = \bar{\epsilon}_a \, (\alpha_2^{(i),+} \cdot \gamma)^a{}_b |\Psi^{(i)}\rangle_A^b, \tag{4.6}$$

$$\delta |\Psi^{(i)}\rangle_A^a = -2 (p^{(i)} \cdot \gamma)^a{}_b \, (\alpha_2^{(i)} \cdot \gamma)^b{}_c \, \epsilon^c \, |\phi^{(i)}\rangle_A \tag{4.7}$$

and then considers their nonlinear deformations by the terms which are compatible with the interactions.[6] One can see, however, that the off-shell transversality conditions (2.10) and (2.18) combined with supersymmetry transformations (4.6)–(4.7) put the fields completely on shell. On the other hand, it is a matter of direct computations to check that the supersymmetry transformations given above transform the vertex (4.3) into (4.4) and vice versa, provided the fields are transversal and obey the massless Klein-Gordon and Dirac equations. This invariance can be explained as follows: in the case of free triplets [11] supersymmetry transformations are generated by the oscillator $\alpha_2^{(i),+}$, see (4.6)–(4.7). Since the fields are on shell, these transformations stay the same also for cubic interactions. Further, both vertices (4.3) and (4.4) have the form of an unknown function which depends only on the oscillators $\alpha_1^{(i),+}$, times prefactors which contain

[6] From these supersymmetry transformations one can see that the fields $\phi_{\nu;\mu_1,\ldots,\mu_n}(x)$ and $\Psi_{\mu_1,\ldots,\mu_n}(x)$ form an $N = 1$ supermultiplet, see [11] for details.

only the oscillators $\alpha_2^{(i),+}$. Therefore, it is sufficient to check how these prefactors transform into each other under the supersymmetry transformations. As one can see, this check repeats exactly the proof for the invariance of cubic interactions in the standard $N = 1$ Super Yang-Mills theory.

4.2. An example: $N = 1$ super Yang-Mills theory

Cubic vertices for $N = 1$ super Yang-Mills theory in $D = 4, 6$ and 10 dimensions are the simplest examples of the ones considered in the previous subsection. We shall consider them in detail also for the purpose of showing how the requirement imposed by supersymmetry for the fields being completely on shell can be lifted by including auxiliary fields, thus promoting the system to an off-shell one.

For the case of super Yang-Mills theory we take at most only one set of oscillators $\alpha_2^{\mu,+}, c_2^+, b_2^+$ in each Fock space, thus making the nonlinear deformation of the Super-Maxwell system considered in [11].

To obtain the Yang-Mills cubic vertex we take the higher spin functional in the form

$$|\phi^{(i)}\rangle_A = \mathcal{A}_{\mu,A}(x)\alpha_2^{\mu(i),+}|0_B^{(i)}\rangle \tag{4.8}$$

and then use (4.3) with the unknown function $\mathcal{F}$ being replaced by a constant

$$|V\rangle_{ABC} = -\frac{ig}{12} f_{ABC}\mathcal{Z}_{222}|0_B^{(1)}\rangle \otimes |0_B^{(2)}\rangle \otimes |0_B^{(3)}\rangle \tag{4.9}$$

In this way one obtains the cubic interaction vertex of Yang-Mills theory

$$V = g f_{ABC}(\partial^\mu \mathcal{A}_A^\nu)\mathcal{A}_{\mu,B}\mathcal{A}_{\nu,C} \tag{4.10}$$

Similarly, we take the higher spin functional in the fermionic sector as

$$|\Psi^{(i)}\rangle_A^a = \Psi_A^a(x)|0_F^{(i)}\rangle \tag{4.11}$$

Then, from the vertex (4.4), with constant $\mathcal{F}$

$$|\mathcal{V}\rangle_{ABC}^{ab} = \frac{ig}{3} f_{ABC}(\alpha_2^{(3),+}\cdot\gamma)^{ab}\ |0^{(1)}\rangle_F \otimes |0^{(2)}\rangle_F \otimes |0^{(3)}\rangle_B + \text{ cyclic} \tag{4.12}$$

we get for the cubic interaction between two fermions and the gauge field

$$\mathcal{V} = ig f_{ABC}\Psi^{a,A}\gamma_{ab}^\mu \Psi^{b,B}\mathcal{A}_\mu^C \tag{4.13}$$

The only nonzero parameter of gauge transformations is

$$|\Lambda_B^{(i)}\rangle^A = ib_2^{(i),+}\lambda^A(x)|0_B^{(i)}\rangle. \tag{4.14}$$

From the equations (C.11) and (C.12) with constant $\mathcal{F}$ we get for the nonzero components of $\mathcal{W}$ vertices

$$|\mathcal{W}_1^{2,3}\rangle_{ABC}^{ab} = |\mathcal{W}_2^{1,3}\rangle_{ABC}^{ab} = f_{ABC}c_2^+ C^{ab}|0_F^{(1)}\rangle \otimes |0_F^{(2)}\rangle \otimes |0_B^{(3)}\rangle, \tag{4.15}$$

which generate the standard gauge transformations for spin $1/2$ fermions in the adjoint representation

$$\delta\Psi_A^a = g f_{ABC}\Psi_B^a\lambda_C \tag{4.16}$$

Then using (4.6)–(4.7) we get the linear part of the standard supersymmetry transformations for the Yang-Mills supermultiplet

$$\delta \mathcal{A}_{\mu,A} = i\Psi^a_A (\gamma_\mu)_{ab}\epsilon^b, \quad \delta\Psi^a_A = i(\gamma^\nu)^a{}_b(\gamma^\mu)^b{}_c\,\epsilon^c\partial_\nu\mathcal{A}_{\mu,A}. \tag{4.17}$$

The equation above describes on-shell vertices. To promote this system off-shell, instead of imposing an off-shell transversality constraint, we introduce an auxiliary $\mathcal{E}^A(x)$ field. That means, that in the bosonic sector we consider a higher spin functional of the form

$$|\Phi^{(i)}_B\rangle^A = (\mathcal{A}^A_\mu(x)\alpha_2^{\mu(i),+} - i\mathcal{E}^A(x)c_0^{(i),+}b_2^{(i),+})|0^{(i)}_B\rangle. \tag{4.18}$$

The expression for the interaction vertex between two fermions and the boson remains unchanged, while the interaction vertex between three bosons we now write as

$$|V\rangle_{ABC} = -\frac{ig}{12}f_{ABC}[(\alpha_2^{(1),+}\cdot\alpha_2^{(2),+})((p^{(1)}-p^{(2)})\cdot\alpha_2^{(3),+} + (b_0^{(1)}-b_0^{(2)})\,c_2^{(3),+})]\times$$
$$\times c_0^{(1)}c_0^{(2)}c_0^{(3)}\;|0^{(1)}_B\rangle\otimes|0^{(2)}_B\rangle\otimes|0^{(3)}_B\rangle + \text{ cyclic} \tag{4.19}$$

The full interacting cubic Lagrangian is a sum of (3.14) and of

$$\mathcal{L}_{\rm int} = \sum_{i=1}^{3}{}^A\langle\Psi^{(i)}|g_0^{(i)}|\Psi^{(i)}\rangle_A + g\left({}^A\langle\Psi^{(1)}_F|{}^B\langle\Psi^{(2)}|{}^C\langle\Phi^{(3)}_B||\mathcal{V}\rangle_{ABC} + \text{ cyclic}\right) \tag{4.20}$$

Using the explicit form of the higher spin functionals in the bosonic (4.18) and fermionic (4.11) sectors, one can see that the auxiliary field $\mathcal{E}^A(x)$ is contained only in the free part of the bosonic Lagrangian. After eliminating it via its own equations of motion one obtains the standard Lagrangian for super Yang-Mills up to the cubic order.

Let us note that the vertex (4.19) generates both the Lagrangian interactions and the nonlinear part of the gauge transformations

$$\delta\mathcal{A}_{\mu,A} = \partial_\mu\lambda_A + gf_{ABC}\mathcal{A}_{\mu,B}\lambda_C. \tag{4.21}$$

The supersymmetry transformations will be deformed with nonlinear terms

$$\delta|\phi^{(i)}\rangle_A = \bar\epsilon_a\,(\alpha_2^{(i),+}\cdot\gamma)^a{}_b|\Psi^{(i)}\rangle^b_A, \tag{4.22}$$

$$\delta|\Psi^{(i)}\rangle^a_A = -2(p^{(i)}\cdot\gamma)^a{}_b\,(\gamma^\mu)^b{}_c\,\alpha^{(i)}_{2,\mu}\,\epsilon^c\,|\phi^{(i)}\rangle_A + \tag{4.23}$$
$$+g\;{}_B\langle\phi^{(i+1)}|_C\langle\phi^{(i+2)}|f_{ABC}(\gamma^{\mu\nu})^a{}_b\alpha^{(i+1),+}_{2,\mu}\alpha^{(i+2),+}_{2,\nu}\epsilon^b\;|0^{(i+1)}_B\rangle\otimes|0^{(i+2)}_B\rangle\otimes|0^{(i)}_F\rangle$$

being the standard supersymmetry transformations for the $N = 1$ Yang-Mills supermultiplet.

The consideration of the Super Yang-Mills theory suggests a very interesting possibility to lift the supersymmetry to an off-shell Lagrangian level, by considering the gauge fixing condition (2.10) only for $m = 1$. In other words, after imposing transversality only with respect to the first set of the indices, the supersymmetry no longer requires the higher spin fields to be on-shell, and since they contain only one $\alpha_2^{(i),+}$ oscillator (like the Yang-Mills vector field), the nonlinear part of the supersymmetry transformations will be the same as in (4.22). Exactly the same arguments can be applied to the supergravity-like systems considered below, with nonlinear parts of supersymmetry transformations being determined by the corresponding $N = 1$ supergravity transformations.

5. Supergravity-like systems

5.1. Vertices

As the second type of the vertices we consider the case where the internal indices are absent. The systems obtained in this way lead to the higher spin generalization of $D = 4$ $N = 1$ supergravity, as we shall see below.

The defining equations and consistency conditions for the supegravity-like vertices are again (3.24)–(3.26) and (3.27)–(3.29). One can see that the corresponding solutions for $\mathcal{W}$ vertices can be obtained from the ones for the vertex (4.1) by simply flipping the signs of $(\mathcal{W}_1^{3,2})^{ab}$, $(\mathcal{W}_2^{1,3})^{ab}$ and $(\mathcal{W}_3^{2,1})^{ab}$, since now we do not have to account for the antisymmetry of the structure constants.

We consider the following interaction vertex between two fermionic and one bosonic fields

$$\langle\phi^{(3)}|\ {}_a\langle\Psi^{(1)}|\ {}_b\langle\Psi^{(2)}|\mathcal{Z}_{111}(\gamma\cdot\alpha_2^{(3),+})^{ab}\mathcal{F}(\mathcal{K}_1^{(i)},\mathcal{Z}_{111})|0_F^{(1)}\rangle\otimes|0_F^{(2)}\rangle\otimes|0_B^{(3)}\rangle + \text{cyclic} \quad (5.1)$$

The cubic vertex for three bosonic fields is

$$\langle\phi^{(1)}|\ \langle\phi^{(2)}|\ \langle\phi^{(3)}|\mathcal{Z}_{111}\mathcal{Z}_{222}\mathcal{F}(\mathcal{K}_1^{(i)},\mathcal{Z}_{111})|0_B^{(1)}\rangle\otimes|0_B^{(2)}\rangle\otimes|0_B^{(3)}\rangle. \quad (5.2)$$

Apart from the absence of internal indices the difference from the super Yang-Mills-like vertex is the inclusion of $\mathcal{Z}_{111}$ in the prefactor. As before, the undetermined arbitrary function in the vertices can depend on $\mathcal{K}_1^{(i)}$ and $\mathcal{Z}_{111}$, but we impose cyclicity using condition (4.2).

The consideration of $N=1$ supersymmetry closely resembles the one for the Super Yang-Mills -like systems. The supersymmetry transformations are (4.6)–(4.7) without internal symmetry indices. They take the vertices (5.1) and (5.2) to each other, with the proof being completely analogous to the one used in Super Yang-Mills. Again the transversality constraint puts the fields completely on-shell.

The generalizing function has to be even

$$\mathcal{F}(-\mathcal{K}_1,-\mathcal{Z}_{111})=\mathcal{F}(\mathcal{K}_1,\mathcal{Z}_{111}) \quad (5.3)$$

otherwise the vertex evaluates to zero because of the symmetry of changing the Fock space labels. This means that the total number of oscillators in the supergravity-like vertices is even.

5.2. *An example: $D=4$, $N=1$ supergravity*

In this section we shall demonstrate how the present approach works for the case of the linearized $D=4$, $N=1$ supergravity.

Following [11], let us take the higher spin functional to contain two oscillators in the bosonic sector

$$|\phi^{(i)}\rangle=\phi_{\mu,\nu}(x)\,\alpha_1^{\mu(i),+}\alpha_2^{\nu(i),+}|0_B^{(i)}\rangle \quad (5.4)$$

with no symmetry between the two indices, and one oscillator in the fermionic sector

$$|\Psi^{(i)}\rangle^a=\Psi^a_\mu(x)\,\alpha_1^{\mu(i),+}|0_F^{(i)}\rangle \quad (5.5)$$

Decomposing the fields into irreducible representations of the Poincaré group as

$$\phi_{\mu,\nu}=\left(\phi_{(\mu,\nu)}-\eta_{\mu\nu}\frac{1}{D}\phi^\rho_\rho\right)+\phi_{[\mu,\nu]}+\eta_{\mu\nu}\frac{1}{D}\phi^\rho_\rho\equiv h_{\mu\nu}+B_{\mu\nu}+\frac{1}{D}\eta_{\mu\nu}\varphi \quad (5.6)$$

and

$$\psi^a_\mu=\Psi^a_\mu+\frac{1}{D}(\gamma^\mu)^{ab}(\gamma^\nu)_{bc}\Psi^c_\nu\equiv\Psi^a_\mu+\frac{1}{D}(\gamma^\mu)^{ab}\,\Xi_b \quad (5.7)$$

one can see, that this field content corresponds to the $D=4$ $N=1$ supergravity supermultiplet and a chiral supermultiplet [68].

The interaction vertices are given by (5.1) and (5.2) with the function $\mathcal{F}$ being a constant. Taking the cubic interaction vertex between three bosons as

$$V = \frac{1}{6} a \mathcal{Z}_{111} \mathcal{Z}_{222}, \tag{5.8}$$

where the expressions for $\mathcal{Z}_{mnp}$ are given in (3.12), one obtains the Lagrangian

$$\mathcal{L}_B = -\phi^{\mu,\nu} \Box \phi_{\mu,\nu} - 4a(\partial_\rho \partial_\tau \phi_{\mu,\nu})\phi^{\mu,\nu}\phi^{\rho,\tau} + 8a(\partial_\rho \partial_\tau \phi_{\mu,\nu})\phi^{\mu,\tau}\phi^{\rho,\nu} \tag{5.9}$$

The vertex describing interactions between two fermionic and one bosonic fields is

$$\frac{f}{3}\left(\langle \phi^{(3)}|_a \langle \Psi^{(1)}|_b \langle \Psi^{(2)}|(\gamma_\mu)^{ab} \alpha_2^{\mu(3),+} \mathcal{Z}_{111}|0_F^{(1)}\rangle \otimes |0_F^{(2)}\rangle \otimes |0_B^{(3)}\rangle + \text{cyclic} \right) \tag{5.10}$$

From these vertices we obtain the Lagrangian for the fermions

$$\mathcal{L}_F = -\frac{1}{2} \bar{\Psi}^\mu \gamma^\nu \partial_\nu \Psi_\mu + 4if\phi^{\mu,\nu} \bar{\Psi}^\alpha \gamma_\nu \partial_\alpha \Psi_\mu - 2if\phi^{\mu,\nu} \bar{\Psi}^\alpha \gamma_\nu \partial_\mu \Psi_\alpha \tag{5.11}$$

One can expand (5.9) and (5.11) and write the Lagrangian in terms of the irreducible components.

In order to consider $N = 1$ supersymmetry, we restrict the fields to be completely on-shell, as we have done for the Yang-Mills like systems. A choice of the constants as $a = -4f$ allows one to match the relative coefficients between the cubic vertices to the one of the linearized pure $D = 4$ $N = 1$ supergravity (see Appendix D for some equations for linearized supergravity). Then one can check that the transformations (4.6)–(4.7) transform the cubic vertices (5.8) and (5.10) into each other. The supersymmetry transformations for irreducible components can be read from (4.6)–(4.7) and correspond to supersymmetry trasnformations of the linearized $D = 4$ $N = 1$ Supergravity [11].

Let us note that the field content (5.6)–(5.7) corresponds also to the irreducible $N = 1$ supergravity supermultiplet in ten dimensions [69]–[70] and to $N = (1, 0)$ gravitational supermultiplet together with $N = (1, 0)$ tensor supermultiplets[7] in six dimensions [71]. However, although the invariance under supersymmetry transformations works exactly in the same way as for four dimensions, a promotion of these higher dimensional models to off-shell ones for higher spin fields might prove problematic. The reason for this is that it does not seem possible to find a consistent higher spin generalization of the vertices which describe the coupling of the $B_{\mu\nu}$ field to the fermions in the $D = 10$, $N = 1$ supergravity [69]–[70] given in equation (D.8). Therefore, an off-shell higher spin extension of higher dimensional supergravity-like models still poses an interesting open problem.

6. Conclusions

In this paper we have constructed the cubic interaction vertices for the massless higher spin supersymmetric theories in four, six and ten dimensions. Our analysis is based on use of the BRST approach to higher spin field theories which works perfectly both for finding the free Lagrangians and the vertices. As a concrete application we have studied the vertices for Yang-Mills-like higher spin theories which are characterized by Lie algebra structure, and for $N = 1$ supegravities in $D = 4$, 6 and 10.

[7] The six-dimensional $N = (1, 0)$ gravitational supermultiplet $(h_{\mu\nu}, B^+_{\mu\nu}, \psi_\mu)$ contains a graviton, the self-dual part of the $B_{\mu\nu}$ field and a chiral gravitino. The six-dimensional $N = (1, 0)$ tensor supermultiplet $(\phi, B^-_{\mu\nu}, \Xi)$ contains a scalar, the anti-self-dual part of the $B_{\mu\nu}$ field and an anti-chiral fermion.

The present paper is a step towards off-shell Lagrangian formulation of supersymmetric higher spin gauge theories in various dimensions. Since computations for the off-shell unconstrained Lagrangians are quite tedious, here we restricted ourselves with the consideration of maximally simplified models. In particular, we started with unconstrained free Lagrangians for massless reducible representations of the Poincaré group and gauge fixed them to contain only d'Alembertian and Dirac operators, while keeping the transversality conditions off-shell. As a second step, we considered the cubic interactions for such Lagrangians. Finally, we showed that supersymmetry transformations, under which the obtained system is invariant, put the fields completely on-shell. All these steps, however, can be generalized to an unconstrained off-shell form via straightforward computations, which we leave to a separate publication. It is interesting to note that in four dimensions the most convenient way to develop the unconstrained formulation is one in terms of two-component totally symmetric spin tensors where the trace conditions are automatically fulfilled (see e.g. [75]–[76]).

It would be interesting to consider massive higher spin supermultiplets in higher dimensions, a topic which to the best of our knowledge has not been yet explored. Further inclusion of cubic interactions into these systems is not only interesting in its own right, but hopefully might shed some new light on the role played by massive higher spin modes in superstring theories (see [77] for a recent study in this direction).

A possible deformation of the models presented in the present paper to curved backgrounds is yet another interesting problem. For this purpose AdS_D space is a natural choice, since it is generically compatible with supersymmetry, unlike de Sitter spaces (see [74] for a recent discussion on higher spin theories on dS_4). Again, despite the recent progress in studies of supersymmetric higher spin models on AdS backgrounds [72]–[73], the higher dimensional generalizations are not known.

Most importantly, it is interesting to find if there is a possibility for building supersymmetric models which have consistent higher order classical, and possibly quantum, interactions. It is well known that real difficulties in higher spin theories start when considering higher order interactions, even at the classical level.[8] It would be very interesting, therefore, to explore the possibility of the existence of supersymmetric theories with massive and/or massless higher spin fields with consistent higher order classical and quantum interactions.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

We are grateful to Yasha Neiman and Dmitri Sorokin for useful discussions. We would like to thank Konstantinos Koutrolikos and Sergei Kuzenko for correspondence and comments. The work of I.L.B. and V.A.K. was partially supported by The Ministry of Education of Russian Federation, project FEWF-2020-0003. The work of M.T. and D.W. was supported by the Quantum Gravity Unit of the Okinawa Institute of Science and Technology Graduate University (OIST).

[8] For the non-supersymmetric case this problem can be overcome by considering four or three dimensional theories in the light-front gauge with the coupling constants in the cubic vertices having a specific form [78–82]. A detailed study of the quantum properties of these models has been performed in [83–85].

Appendix A. Conventions

We mainly follow the notations of [86], where some more useful identities for spinors and for gamma matrices can be found.

Throughout the paper "(,)" denotes symmetrization and "[,]" denotes antisymmetrization with weight one. The Latin letters $a, b \dots$ label spinorial indices. The Greek letters $\mu, \nu, \dots$ label flat space-time vector indices and Greek letters with "hat" $\hat{\mu}, \hat{\nu}, \dots$ label vector indices in curved space-time.

We choose a real representation for Majorana spinors

$$(\lambda^a)^\star = \lambda^a, \quad \bar{\lambda}_a = \lambda^b C_{ba} \tag{A.1}$$

The spinor indices can be raised and lowered by anti-symmetric charge conjugation matrices C_{ab} and C^{ab} as

$$\lambda^a = C^{ab}\lambda_b, \quad \lambda_a = \lambda^b C_{ba}, \quad C^{ab}C_{bc} = -\delta^a_c. \tag{A.2}$$

The γ–matrices satisfy the following anti-commutation relations

$$(\gamma^\mu)^a{}_c(\gamma^\nu)^c{}_b + (\gamma^\nu)^a{}_c(\gamma^\mu)^c{}_b = 2\eta^{\mu\nu}\delta^a_b. \tag{A.3}$$

In $D = 4$ the matrices γ_μ and $\gamma_{\mu\nu}$ with both spinorial indices up (down) are symmetric and the matrices C, γ_5 and $\gamma_5\gamma_\mu$ are antisymmetric. In $D = 10$ the matrices γ_μ and $\gamma_{\mu_1,\dots,\mu_5}$ with both spinorial indices up (down) are symmetric, and the matrices $\gamma_{\mu_1\mu_2\mu_3}$ are antisymmetric.

For checking the on-shell closure of the supersymmetry algebra and of the supersymmetry of the vertices we have used the following gamma-matrix identities

$$(\gamma^\nu)_{ab}(\gamma_\nu)_{cd} + (\gamma^\nu)_{ac}(\gamma_\nu)_{db} + (\gamma^\nu)_{ad}(\gamma_\nu)_{bc} = 0, \tag{A.4}$$

$$\gamma^\mu\gamma^{\nu_1,\nu_2,\dots\nu_r}\gamma_\mu = (-1)^r(D-2r)\gamma^{\nu_1,\nu_2,\dots\nu_r}. \tag{A.5}$$

For a product of gamma matrices we have

$$\gamma^{\nu_1,\dots,\nu_i}\gamma_{\mu_1,\dots,\mu_j} = \sum_{k=0}^{k=min(i,j)} \frac{i!j!}{(i-k)!(j-k)!k!}\gamma^{[\nu_1,\dots,\nu_{i-k}}{}_{[\mu_{k+1},\dots,\mu_j}\delta^{\nu_i}_{\mu_1}\delta^{\nu_{i-1}}_{\mu_2}\dots\delta^{\nu_{n-k+1}]}_{\mu_k]} \tag{A.6}$$

and in particular

$$\gamma_{\mu\nu\rho} = \gamma_{\mu\nu}\gamma_\rho - 2\eta_{\rho[\nu}\gamma_{\mu]}. \tag{A.7}$$

Appendix B. Equations for $\mathcal{W}$ vertices

The equations which express the $\mathcal{W}$ vertices in terms of the $\mathcal{V}$ vertices present in the Lagrangian (3.15) are

$$\int dc_0^{(3)}\left(Q^{(3)}|\mathcal{V}_{11}\rangle^{ab}_{ABC} - \frac{1}{\sqrt{2}}(g_0^{(1)})^a{}_c|\mathcal{W}^{2,3}_{1,11}\rangle^{cb}_{BCA} + \frac{1}{\sqrt{2}}(g_0^{(2)})^b{}_c|\mathcal{W}^{1,3}_{2,11}\rangle^{ca}_{ACB} + \right.$$
$$\left. - \tilde{Q}^{(1)}|\mathcal{W}^{2,3}_{1,21}\rangle^{ab}_{BCA} + \tilde{Q}^{(2)}|\mathcal{W}^{1,3}_{2,21}\rangle^{ba}_{ACB}\right) = 0 \tag{B.1}$$

$$\int dc_0^{(3)} \Big(Q^{(3)}|\mathcal{V}_{22}\rangle^{ab}_{ABC} + \sqrt{2}M_F^{(1)}(g_0^{(1)})^a{}_c|\mathcal{W}^{2,3}_{1,22}\rangle^{cb}_{ACB} + \sqrt{2}M_F^{(2)}(g_0^{(2)})^b{}_c|\mathcal{W}^{1,3}_{2,22}\rangle^{ca}_{BCA} + $$
$$+ \tilde{Q}^{(1)}|\mathcal{W}^{2,3}_{1,12}\rangle^{ab}_{BCA} + \tilde{Q}^{(2)}|\mathcal{W}^{1,3}_{2,12}\rangle^{ba}_{ACB}\Big) = 0 \tag{B.2}$$

$$\int dc_0^{(3)} \Big(Q^{(3)}|\mathcal{V}_{12}\rangle^{ab}_{ABC} + \frac{1}{\sqrt{2}}(g_0^{(1)})^a{}_c|\mathcal{W}^{2,3}_{1,12}\rangle^{cb}_{BCA} - \sqrt{2}M_F^{(2)}(g_0^{(2)})^b{}_c|\mathcal{W}^{1,3}_{2,21}\rangle^{ca}_{ACB} + $$
$$- \tilde{Q}^{(1)}|\mathcal{W}^{2,3}_{1,22}\rangle^{ab}_{BCA} + \tilde{Q}^{(2)}|\mathcal{W}^{1,3}_{2,11}\rangle^{ba}_{ACB}\Big) = 0 \tag{B.3}$$

$$\int dc_0^{(3)} \Big(Q^{(3)}|\mathcal{V}_{21}\rangle^{ab}_{ABC} + \frac{1}{\sqrt{2}}(g_0^{(2)})^b{}_c|\mathcal{W}^{1,3}_{2,12}\rangle^{ca}_{ACB} - \sqrt{2}M_F^{(1)}(g_0^{(1)})^a{}_c|\mathcal{W}^{2,3}_{1,21}\rangle^{cb}_{BCA} + $$
$$+ \tilde{Q}^{(1)}|\mathcal{W}^{2,3}_{1,11}\rangle^{ab}_{BCA} - \tilde{Q}^{(2)}|\mathcal{W}^{1,3}_{2,22}\rangle^{ba}_{ACB}\Big) = 0 \tag{B.4}$$

$$\int dc_0^{(3)} \Big(\tilde{Q}_F^{(1)}|\mathcal{V}_{11}\rangle^{ab}_{ABC} + \frac{1}{\sqrt{2}}(g_0^{(1)})^a{}_c|\mathcal{V}_{21}\rangle^{cb}_{ABC} + Q_B^{(3)}|\mathcal{W}^{2,1}_{3,11}\rangle^{ba}_{BAC} + $$
$$+ \frac{1}{\sqrt{2}}(g_0^{(2)})^b{}_c|\mathcal{W}^{3,1}_{2,11}\rangle^{ca}_{CAB} - \tilde{Q}_F^{(2)}|\mathcal{W}^{3,1}_{2,21}\rangle^{ba}_{CAB}\Big) = 0 \tag{B.5}$$

$$\int dc_0^{(3)} \Big(\tilde{Q}_F^{(1)}|\mathcal{V}_{12}\rangle^{ab}_{ABC} - \frac{1}{\sqrt{2}}(g_0^{(1)})^a{}_c|\mathcal{V}_{22}\rangle^{cb}_{ABC} + Q_B^{(3)}|\mathcal{W}^{2,1}_{3,21}\rangle^{ba}_{BAC} + $$
$$+ \sqrt{2}M_F^{(2)}(g_0^{(2)})^b{}_c|\mathcal{W}^{3,1}_{2,21}\rangle^{ca}_{CAB} - \tilde{Q}_F^{(2)}|\mathcal{W}^{3,1}_{2,11}\rangle^{ba}_{CAB}\Big) = 0 \tag{B.6}$$

$$\int dc_0^{(3)} \Big(\tilde{Q}_F^{(1)}|\mathcal{V}_{21}\rangle^{ab}_{ABC} + \sqrt{2}M_F^{(1)}(g_0^{(1)})^a{}_c|\mathcal{V}_{11}\rangle^{cb}_{ABC} + Q_B^{(3)}|\mathcal{W}^{2,1}_{3,12}\rangle^{ba}_{BAC} + $$
$$+ \frac{1}{\sqrt{2}}(g_0^{(2)})^b{}_c|\mathcal{W}^{3,1}_{2,12}\rangle^{ca}_{CAB} + \tilde{Q}_F^{(2)}|\mathcal{W}^{3,1}_{2,22}\rangle^{ba}_{CAB}\Big) = 0 \tag{B.7}$$

$$\int dc_0^{(3)} \Big(\tilde{Q}_F^{(1)}|\mathcal{V}_{22}\rangle^{ab}_{ABC} - \sqrt{2}M_F^{(1)}(g_0^{(1)})^a{}_c|\mathcal{V}_{12}\rangle^{cb}_{ABC} + Q_B^{(3)}|\mathcal{W}^{2,1}_{3,22}\rangle^{ba}_{BAC} + $$
$$- \sqrt{2}M_F^{(2)}(g_0^{(2)})^b{}_c|\mathcal{W}^{3,1}_{2,22}\rangle^{ca}_{CAB} - \tilde{Q}_F^{(2)}|\mathcal{W}^{3,1}_{2,12}\rangle^{ba}_{CAB}\Big) = 0 \tag{B.8}$$

$$\int dc_0^{(3)} \Big(\tilde{Q}_F^{(2)}|\mathcal{V}_{11}\rangle^{ab}_{ABC} - \frac{1}{\sqrt{2}}(g_0^{(2)})^b{}_c|\mathcal{V}_{12}\rangle^{ac}_{ABC} - Q_B^{(3)}|\mathcal{W}^{1,2}_{3,11}\rangle^{ab}_{ABC} + $$
$$- \frac{1}{\sqrt{2}}(g_0^{(1)})^a{}_c|\mathcal{W}^{3,2}_{1,11}\rangle^{cb}_{CBA} + \tilde{Q}_F^{(1)}|\mathcal{W}^{3,2}_{1,21}\rangle^{ab}_{CBA}\Big) = 0 \tag{B.9}$$

$$\int dc_0^{(3)} \Big(\tilde{Q}_F^{(2)}|\mathcal{V}_{21}\rangle^{ab}_{ABC} - \frac{1}{\sqrt{2}}(g_0^{(2)})^b{}_c|\mathcal{V}_{22}\rangle^{ac}_{ABC} + Q_B^{(3)}|\mathcal{W}^{1,2}_{3,21}\rangle^{ab}_{ABC} + $$
$$+ \sqrt{2}M_F^{(1)}(g_0^{(1)})^a{}_c|\mathcal{W}^{3,2}_{1,21}\rangle^{cb}_{CBA} - \tilde{Q}_F^{(1)}|\mathcal{W}^{3,2}_{1,11}\rangle^{ab}_{CBA}\Big) = 0 \tag{B.10}$$

$$\int dc_0^{(3)} \Big(\tilde{Q}_F^{(2)}|\mathcal{V}_{12}\rangle^{ab}_{ABC} - \sqrt{2}M_F^{(2)}(g_0^{(2)})^b{}_c|\mathcal{V}_{11}\rangle^{ac}_{ABC} + Q_B^{(3)}|\mathcal{W}^{1,2}_{3,12}\rangle^{ab}_{ABC} + $$
$$+ \frac{1}{\sqrt{2}}(g_0^{(1)})^b{}_c|\mathcal{W}^{3,2}_{1,12}\rangle^{cb}_{CBA} + \tilde{Q}_F^{(1)}|\mathcal{W}^{3,2}_{1,22}\rangle^{ab}_{CBA}\Big) = 0 \tag{B.11}$$

$$\int dc_0^{(3)} \left(\tilde{Q}_F^{(2)} |\mathcal{V}_{22}\rangle^{ab}_{ABC} - \sqrt{2} M_F^{(2)} (g_0^{(2)})^b{}_c |\mathcal{V}_{12}\rangle^{ac}_{ABC} + Q_B^{(3)} |\mathcal{W}^{1,2}_{3,22}\rangle^{ab}_{ABC} + \right.$$
$$\left. - \sqrt{2} M_F^{(1)} (g_0^{(1)})^a{}_c |\mathcal{W}^{3,2}_{1,22}\rangle^{cb}_{CBA} - \tilde{Q}_F^{(1)} |\mathcal{W}^{3,2}_{1,12}\rangle^{ab}_{CBA} \right) = 0 \tag{B.12}$$

The ghost numbers for each vertex and of each parameter of the gauge transformations can be easily deduced, using the following counting: the ghost number of the BRST charges is equal to $+1$, integration over the ghost zero mode $c_0^{(3)}$ carries the ghost number -1, the Lagrangian has the ghost number zero. Then, the free part of (3.15) implies that the fields $|\Phi^{(i)}_{F,1}\rangle^b_A$ and $|\Phi^{(i)}_B\rangle_A$ have ghost number zero and the field $|\Phi^{(i)}_{F,2}\rangle^b_A$ has ghost number -1. Similarly, the gauge transformation rules (3.17)–(3.19) and the cubic part of the Lagrangian (3.15) determine the ghost numbers of the parameters of gauge transformations and of the vertices.

Of the above twelve equations, seven have a unique form. The remaining five are related to the others by utilizing the symmetry exchanging the Hilbert space labels for the fermions $|\Phi^{(1)}_F\rangle$ and $|\Phi^{(2)}_F\rangle$, together with the appropriate transformation of the vertex:

$$1 \leftrightarrow 2\,, \qquad |\mathcal{V}_{mn}\rangle^{ab}_{ABC} \to (-1)^{mn} |\mathcal{V}_{nm}\rangle^{ba}_{BAC}. \tag{B.13}$$

This operation will take the equation (B.3) to the equation (B.4) and, respectively, the equations (B.5)-(B.8) to the equations (B.9)-(B.12).

The system of equations is reduced to their gauge fixed form of equations (3.24)-(3.26) by taking

$$|\mathcal{V}_{11}\rangle = c_0^{(3)} |\mathcal{V}\rangle, \tag{B.14}$$
$$|\mathcal{W}^{j,3}_{i,11}\rangle = \sqrt{2} c_0^{(3)} |\mathcal{W}^{j,3}_i\rangle, \qquad |\mathcal{W}^{3,j}_{i,11}\rangle = \sqrt{2} c_0^{(3)} |\mathcal{W}^{3,j}_i\rangle, \qquad |\mathcal{W}^{i,j}_{3,11}\rangle = |\mathcal{W}^{i,j}_3\rangle \tag{B.15}$$

and putting the remaining terms equal to zero.

Appendix C. Expressions for $\mathcal{W}$ and $\mathcal{X}$ vertices for super Yang-Mills-like systems

In this appendix we present the solutions for $\mathcal{W}$ and $\mathcal{X}$ vertices for super Yang-Mills-like systems. Analogous solutions for supergravity-like systems can be obtained from the present ones by using the symmetry properties of the defining equations and of the ansatz of the vertex, as explained in section 5.

The vertex we solve for is defined in equation (4.1). In the following we omit the subscript '1' from Z_{111} and $K_1^{(i)}$, and we do not write explicitly the index '3' for the oscillator $\alpha_2^{\mu,+}$, since it is present only in the third Fock space.

Omitting the structure constants, and using the relations

$$\tilde{Q}_B^{(3)} (\mathcal{V})^{ab} = c_2^+ (p^{(3)} \cdot \gamma)^{ab} \mathcal{F} + \tag{C.1}$$
$$+ c_1^{(3),+} (\alpha_2^+ \cdot \gamma)^{ab} ((p^{(2)})^2 - (p^{(1)})^2) \left(\frac{\partial \mathcal{F}}{\partial \mathcal{Z}} (\alpha_1^{(1),+} \cdot \alpha_1^{(2),+}) + \frac{\partial \mathcal{F}}{\partial \mathcal{K}^{(3)}} \right)$$

$$\tilde{Q}_F^{(1)} (\mathcal{V})^{ab} = \tag{C.2}$$
$$= c_1^{(1),+} (\alpha_2^+ \cdot \gamma)^{ab} ((p^{(3)})^2 - (p^{(2)})^2) \left(\frac{\partial \mathcal{F}}{\partial \mathcal{Z}} (\alpha_1^{(2),+} \cdot \alpha_1^{(3),+}) + \frac{\partial \mathcal{F}}{\partial \mathcal{K}^{(1)}} \right)$$

$$\tilde{Q}_F^{(2)}(\mathcal{V})^{ab} = \tag{C.3}$$
$$= c_1^{(2),+}(\alpha_2^+ \cdot \gamma)^{ab}((p^{(1)})^2 - (p^{(3)})^2)\left(\frac{\partial \mathcal{F}}{\partial \mathcal{Z}}(\alpha_1^{(3),+} \cdot \alpha_1^{(1),+}) + \frac{\partial \mathcal{F}}{\partial \mathcal{K}^{(2)}}\right)$$

one can solve the equations

$$(g_0^{(1)})^a{}_b|\mathcal{W}_1^{2,3}\rangle^{bc} + (g_0^{(2)})^c{}_b|\mathcal{W}_2^{1,3}\rangle^{ba} + \tilde{Q}_B^{(3)}|\mathcal{V}\rangle^{ac} = 0 \tag{C.4}$$
$$(g_0^{(1)})^a{}_b|\mathcal{W}_1^{3,2}\rangle^{bc} - l_0^{(3)}|\mathcal{W}_3^{1,2}\rangle^{ac} - \tilde{Q}_F^{(2)}|\mathcal{V}\rangle^{ac} = 0 \tag{C.5}$$
$$(g_0^{(2)})^a{}_b|\mathcal{W}_2^{3,1}\rangle^{bc} - l_0^{(3)}|\mathcal{W}_3^{2,1}\rangle^{ac} - \tilde{Q}_F^{(1)}|\mathcal{V}\rangle^{ca} = 0 \tag{C.6}$$

to get for $\mathcal{W}$-vertices

$$(\mathcal{W}_3^{1,2})^{ab} = c_1^{(2),+}(\gamma \cdot \alpha_2^+)^{ab}\left(\frac{\partial \mathcal{F}}{\partial \mathcal{Z}}(\alpha_1^{(3),+} \cdot \alpha_1^{(1),+}) + \frac{\partial \mathcal{F}}{\partial \mathcal{K}^{(2)}}\right) \tag{C.7}$$

$$(\mathcal{W}_3^{2,1})^{ab} = -c_1^{(1),+}(\gamma \cdot \alpha_2^+)^{ab}\left(\frac{\partial \mathcal{F}}{\partial \mathcal{Z}}(\alpha_1^{(2),+} \cdot \alpha_1^{(3),+}) + \frac{\partial \mathcal{F}}{\partial \mathcal{K}^{(1)}}\right) \tag{C.8}$$

$$(\mathcal{W}_1^{3,2})^{ab} = c_1^{(2),+}(p^{(1)} \cdot \gamma)^a{}_c(\gamma \cdot \alpha_2^+)^{cb}\left(\frac{\partial \mathcal{F}}{\partial \mathcal{Z}}(\alpha_1^{(3),+} \cdot \alpha_1^{(1),+}) + \frac{\partial \mathcal{F}}{\partial \mathcal{K}^{(2)}}\right) \tag{C.9}$$

$$(\mathcal{W}_2^{3,1})^{ab} = -c_1^{(1),+}(p^{(2)} \cdot \gamma)^a{}_c(\gamma \cdot \alpha_2^+)^{cb}\left(\frac{\partial \mathcal{F}}{\partial \mathcal{Z}}(\alpha_1^{(2),+} \cdot \alpha_1^{(3),+}) + \frac{\partial \mathcal{F}}{\partial \mathcal{K}^{(1)}}\right) \tag{C.10}$$

$$(\mathcal{W}_1^{2,3})^{ab} = -c_2^+ C^{ab}\mathcal{F} + \tag{C.11}$$
$$+ c_1^{(3),+}(p^{(1)} \cdot \gamma)^a{}_c(\gamma \cdot \alpha_2^+)^{cb}\left(\frac{\partial \mathcal{F}}{\partial \mathcal{Z}}(\alpha_1^{(1),+} \cdot \alpha_1^{(2),+}) + \frac{\partial \mathcal{F}}{\partial \mathcal{K}^{(3)}}\right)$$

$$(\mathcal{W}_2^{1,3})^{ab} = -c_2^+ C^{ab}\mathcal{F} - \tag{C.12}$$
$$- c_1^{(3),+}(p^{(2)} \cdot \gamma)^a{}_c(\gamma \cdot \alpha_2^+)^{cb}\left(\frac{\partial \mathcal{F}}{\partial \mathcal{Z}}(\alpha_1^{(1),+} \cdot \alpha_1^{(2),+}) + \frac{\partial \mathcal{F}}{\partial \mathcal{K}^{(3)}}\right)$$

Similarly, from the requirement of preservation of group structure for gauge transformations (3.27)–(3.29) we get

$$\tilde{Q}_F^{(2)}|\mathcal{W}_1^{2,3}\rangle^{ab} - \tilde{Q}_B^{(3)}|\mathcal{W}_1^{3,2}\rangle^{ab} = \tilde{Q}_F^{(1)}|\mathcal{X}_1\rangle^{ab} \tag{C.13}$$
$$\tilde{Q}_F^{(1)}|\mathcal{W}_2^{1,3}\rangle^{ab} - \tilde{Q}_B^{(3)}|\mathcal{W}_2^{3,1}\rangle^{ab} = \tilde{Q}_F^{(2)}|\mathcal{X}_2\rangle^{ab} \tag{C.14}$$
$$\tilde{Q}_F^{(1)}|\mathcal{W}_3^{1,2}\rangle^{ab} + \tilde{Q}_F^{(2)}|\mathcal{W}_3^{2,1}\rangle^{ba} = \tilde{Q}_B^{(3)}|\mathcal{X}_3\rangle^{ab} \tag{C.15}$$

Using the solutions (C.7) – (C.12) one can solve for $\mathcal{X}$-vertices,

$$\mathcal{X}_1^{ab} = c_1^{(2),+}c_2^+ b_1^{(1),+}C^{ab}\frac{\partial \mathcal{F}}{\partial \mathcal{Z}}(p^{(1)} \cdot \alpha_1^{(3),+}) + \tag{C.16}$$
$$+ c_1^{(2),+}c_1^{(3),+}b_1^{(1),+}(p^{(1)} \cdot \gamma)^a{}_c(\gamma \cdot \alpha_2^+)^{cb} \times$$
$$\times [-\frac{\partial \mathcal{F}}{\partial \mathcal{Z}} + \frac{\partial^2 \mathcal{F}}{\partial \mathcal{Z}\partial \mathcal{K}^{(2)}}(p^{(1)} \cdot \alpha_1^{(2),+}) - \frac{\partial^2 \mathcal{F}}{\partial \mathcal{Z}\partial \mathcal{K}^{(3)}}(p^{(1)} \cdot \alpha_1^{(3),+}) +$$
$$+ \frac{\partial^2 \mathcal{F}}{\partial \mathcal{Z}^2}(-(\alpha_1^{(1)+} \cdot \alpha_1^{(2),+})(p^{(1)} \cdot \alpha_1^{(3),+}) + (\alpha_1^{(3)+} \cdot \alpha_1^{(1),+})(p^{(1)} \cdot \alpha_1^{(2),+}))]$$

$$
\begin{aligned}
\mathcal{X}_2^{ab} &= -c_2^+ c_1^{(1),+} b_1^{(2),+} C^{ab} \frac{\partial \mathcal{F}}{\partial \mathcal{Z}} (p^{(2)} \cdot \alpha_1^{(3),+}) - \\
&- c_1^{(3),+} c_1^{(1),+} b_1^{(2),+} (p^{(2)} \cdot \gamma)^a{}_c (\gamma \cdot \alpha_2^+)^{cb} \times \\
&\times [-\frac{\partial \mathcal{F}}{\partial \mathcal{Z}} + \frac{\partial^2 \mathcal{F}}{\partial \mathcal{Z} \partial \mathcal{K}^{(3)}} (p^{(2)} \cdot \alpha_1^{(3),+}) - \frac{\partial^2 \mathcal{F}}{\partial \mathcal{Z} \partial \mathcal{K}^{(1)}} (p^{(2)} \cdot \alpha_1^{(1),+}) + \\
&+ \frac{\partial^2 \mathcal{F}}{\partial \mathcal{Z}^2} (-(\alpha_1^{(2)+} \cdot \alpha_1^{(3),+})(p^{(2)} \cdot \alpha_1^{(1),+}) + (\alpha_1^{(1)+} \cdot \alpha_1^{(2),+})(p^{(2)} \cdot \alpha_1^{(3),+}))]
\end{aligned}
\tag{C.17}
$$

$$
\begin{aligned}
\mathcal{X}_3^{ab} &= c_1^{(1),+} c_1^{(2),+} b_1^{(3)+} (\gamma \cdot \alpha_2^+)^{ab} \times \\
&\times [-\frac{\partial \mathcal{F}}{\partial \mathcal{Z}} + \frac{\partial^2 \mathcal{F}}{\partial \mathcal{Z} \partial \mathcal{K}^{(1)}} (p^{(3)} \cdot \alpha_1^{(1),+}) - \frac{\partial^2 \mathcal{F}}{\partial \mathcal{Z} \partial \mathcal{K}^{(2)}} (p^{(3)} \cdot \alpha_1^{(2),+}) + \\
&+ \frac{\partial^2 \mathcal{F}}{\partial \mathcal{Z}^2} (-(\alpha_1^{(3)+} \cdot \alpha_1^{(1),+})(p^{(3)} \cdot \alpha_1^{(2),+}) + (\alpha_1^{(3)+} \cdot \alpha_1^{(2),+})(p^{(3)} \cdot \alpha_1^{(1),+}))]
\end{aligned}
\tag{C.18}
$$

This completes our treatment of the cubic vertices for the Super Yang-Mills like systems.

Appendix D. Some expressions for linearized gravity

In this section we collect some expressions for linearized gravity, which we use for extracting the cubic part from the supergravity Lagrangian.

The metric, the vierbein, the spin connection, and Christoffel symbols are

$$g_{\hat\mu\hat\nu} = \eta_{\hat\mu\hat\nu} + h_{\hat\mu\hat\nu}, \quad g^{\hat\mu\hat\nu} = \eta^{\hat\mu\hat\nu} - h^{\hat\mu\hat\nu}, \tag{D.1}$$

$$e^{\mu}_{\hat\mu} = \delta^{\mu}_{\hat\mu} + \frac{1}{2} h^{\mu}_{\hat\mu}, \quad e^{\hat\mu}_{\mu} = \delta^{\hat\mu}_{\mu} - \frac{1}{2} h^{\hat\mu}_{\mu}, \tag{D.2}$$

$$\omega_{\hat\mu}{}^{\nu\rho} = -\frac{1}{2}(\partial^\nu h^\rho_{\hat\mu} - \partial^\rho h^\nu_{\hat\mu}), \tag{D.3}$$

$$\Gamma^{\hat\rho}_{\hat\mu\hat\nu} = \frac{1}{2}\eta^{\hat\rho\hat\lambda}(\partial_{\hat\mu} h_{\hat\lambda\hat\nu} + \partial_{\hat\nu} h_{\hat\lambda\hat\mu} - \partial_{\hat\lambda} h_{\hat\mu\hat\nu}) \tag{D.4}$$

The Ricci tensor reads

$$R_{\hat\mu\hat\nu} = -\frac{1}{2}(\Box h_{\hat\mu\hat\nu} - \partial_{\hat\mu}\partial_{\hat\lambda} h^{\hat\lambda}_{\hat\nu} - \partial_{\hat\nu}\partial_{\hat\lambda} h^{\hat\lambda}_{\hat\mu} + \partial_{\hat\mu}\partial_{\hat\nu} h^{\hat\lambda}_{\hat\lambda}) \tag{D.5}$$

In the equations above the indices are raised and lowered using the flat metric $\eta_{\hat\mu\hat\nu}$. The covariant derivative acting on vectors and spin–vectors is defined as follows

$$\nabla_{\hat\mu} A_{\hat\nu} = \partial_{\hat\mu} A_{\hat\nu} - \Gamma^{\hat\lambda}_{\hat\mu\hat\nu} A_{\hat\lambda} \tag{D.6}$$

$$\nabla_{\hat\mu} \Psi^a_{\hat\nu} = D_{\hat\mu} \Psi^a_{\hat\nu} - \Gamma^{\hat\lambda}_{\hat\mu\hat\nu} \Psi^a_{\hat\lambda}, \quad D_{\hat\mu} \Psi^a_{\hat\nu} = \partial_{\hat\mu} \Psi^a_{\hat\nu} + \frac{1}{4}\omega_{\hat\mu}{}^{\rho\sigma}(\gamma_{\rho\sigma}\Psi_{\hat\nu})^a \tag{D.7}$$

The vertices coupling the fermions to the B-field in ten dimensional $N = 1$ supergravity mentioned at the end of section 5.2 are

$$\sqrt{2}(\bar\Psi_\mu \gamma^{\mu\tau\sigma\lambda\nu}\Psi_\nu + \bar\Psi^\tau \gamma^\sigma \Psi^\lambda)\partial_\tau B_{\sigma\lambda} - 2(\bar\Psi_\mu \gamma^{\tau\sigma\lambda}\gamma^\mu \Xi)\partial_\tau B_{\sigma\lambda} \tag{D.8}$$

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