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ORIGINAL PAPER

Bennu and Ryugu: diamonds in the sky

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Abstract

Rapidly spinning and loosely aggregated asteroids appear to take on diamond-shaped profiles, with elevated poles as well as equators. The evolutionary processes that form these characteristic shapes remain a matter of debate. In this paper, we propose a novel model, based on debris accretion, to explain these diamond-shaped profiles. We derive an analytic expression for the shapes of such rapidly spinning rubble piles based on the principle that as rubble is deposited it assumes a critical angle of repose. We show that this expression correctly reproduces diamond shaped profiles. We also conduct granular simulations of debris deposition and show that simulated shapes are in striking accord with both observations and analytical results. Our results suggest that non-uniform debris accumulation, which is overlooked in current models, may play a cardinal role in the formation of diamond-shaped asteroids.

Keywords Rubble pile asteroids · Rotating asteroids · Granular material · Discrete element method

Recent visits by JAXA and NASA spacecraft to near-earth asteroids 101,955 Bennu and 162,173 Ryugu have revealed similar diamond-shaped profiles, with higher elevations at both equator and poles [1, 2] (Fig. 1a, b). Both asteroids are rapidly rotating and loosely aggregated rubble-piles [3]. Most current models attribute these characteristic shapes to the redistribution of aggregated material by Yarkovsky-O'Keefe-Radzievskii-Paddack (YORP) induced rotation, which drives material from the poles to the equator due to centrifugal forces [4–8]. These simulations, however, result in flattened and sometimes asymmetric shapes that do not conform to the observed shapes of Ryugu and Bennu [1, 2].

Further, recent analyses of geologic features on Ryugu and Bennu suggest that these asteroids obtained their characteristic shapes at early times, which is at odds with later, YORP-driven, processes [9, 10]. Additional studies have therefore explored whether such shapes might emerge directly from violent fragmentation and subsequent coalescence of fragments of a parent body [10, 11]. Those simulations also fall short of generating diamond-shaped asteroids, and instead typically result in rapidly spinning oblate spheroids. This again suggests that other processes are involved in the evolution of a diamond shape.

To explain the discrepancies in aspect ratio and shape, we note that rubble pile asteroids do not develop from spinning spheres, but form by accretion of debris, which involves distinct morphogenic processes. In particular, centrifugal forces decrease toward the poles, which causes material to accumulate there. We show that this accumulation results in increased elevations at the poles, while centrifugal migration increases elevations at the equator. Without this accumulation, poles lack the elevated poles that are evident on Ryugu and Bennu. We can estimate the effect of changes in centrifugal forces with latitude by considering a block of debris on a rotating surface as sketched in Fig. 2a. The forces on the block shown in the sketch depend on the angle of the surface, $\hat{\vartheta}$, with respect to the horizontal and the position $\beta = \mathrm{atan}(z/r)$, where (r, z) are radius and height in cylindrical coordinates (Fig. 2b). In the Supplementary Information, we show that this expression can be extended to geometries rotating at speed ω to yield an angle of repose of the free surface [12–14]:

$$\hat{\vartheta} = \tan^{-1}\left[\frac{\mu\sin\beta + \cos\beta - \alpha(\mathrm{r},\mathrm{z})}{-\mu\cos\beta + \sin\beta + \mu\alpha(\mathrm{r},\mathrm{z})}\right]. \quad (1)$$

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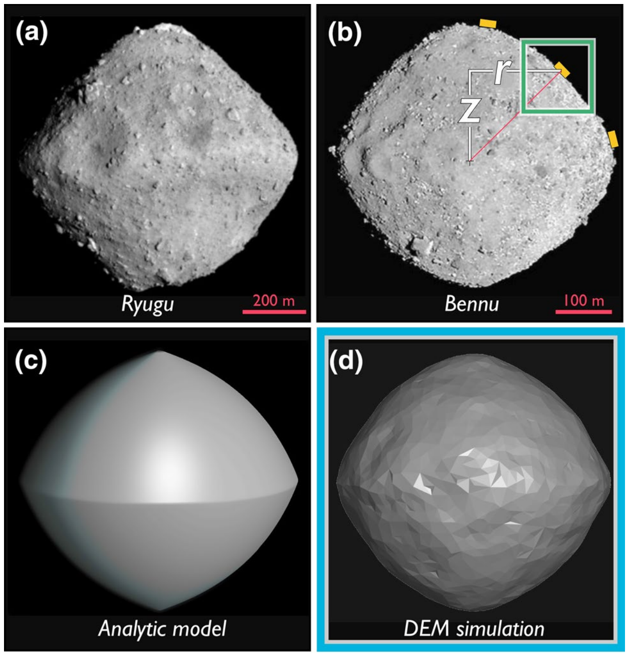

Fig. 1 Diamond profile asteroids. Near-earth asteroids **a** Ryugu and **b** Bennu. Yellow blocks added to panel **b** suggest accumulated material: note that material nearer the poles feels weaker centrifugal force than material at mid- or low- latitudes. Region in green square is enlarged in Fig. 2a. **c** Analytic prediction from Eq. 1 using $\alpha = 0.999$, $\mu = 0.4$. **d** Exterior surface defined by discrete element simulation described in text. Image credits Ryugu: JAXA, Bennu: NASA

Here μ is a friction coefficient (see discussion below), and $\alpha(r,z) = \frac{\omega^2 R_{eq}}{g_{eq}} \frac{rR^2}{R_{eq}^3} = \frac{\omega^2 r R^2}{g_{eq} R_{eq}^2}$ is the ratio between centrifugal and gravitational forces, where R_{eq} and g_{eq} are respectively the radius and gravity at the equator. We take gravity to always be centrally directed, with distance from surface to center $R = \sqrt{r^2 + z^2}$, and by simple substitution at the equator the ratio of outward to inward forces is $\alpha_{eq} = \frac{\omega^2 R_{eq}}{g_{eq}}$.

Equation 1 defines the angle of the surface at every point, and so starting at the equator, $(r,z) = \left(R_{eq}, 0\right)$, we can use this equation to predict the body's shape by advancing at successive increments of latitude toward the pole, as shown in Fig. 1c. Bennu [2] and Ryugu [1] are believed to have formed at very high rotation rates, so for this figure we use the maximum value, $\alpha_{eq} \sim 1$ (technically the arctangent is ill-defined $\alpha = 1$, so in Fig. 1c we use $\alpha_{eq} = 0.999$).

The friction coefficient is not simple to assign, as mixed-shape rocks can interlock (increasing μ) or abruptly tumble (decreasing it), but literature [15] on asteroids often uses a sliding coefficient of 0.6 and a rolling coefficient much smaller [16]. In Fig. 1c, we use $\mu = 0.4$. We find that μ has little effect on the qualitative shape shown, though pole-to-equator ratio grows with μ. Smaller values of α_{eq} also increase the pole-to-equator ratio, and generate equatorial valleys as well. We believe these valleys may affect subsurface texture near the equator, but are otherwise unphysical because Eq. 1 defines the angle of repose, but does not account for conservation of transported mass. At $\alpha_{eq} = 1$, excess mass that arrives at the equator will be flung into space; for smaller values of α_{eq}, mass will flow to the equator where it can be expected to fill in any incipient valley until it reaches a height where $\alpha_{eq} = 1$.

The angle of repose given by Eq. 1 is a convenient simplification, and to assess whether this angle is adopted during the evolution of rapidly rotating rubble piles, we perform simulations of the deposition of particles on a rigid rotating spherical core (Fig. 2b). The presence of an underlying stiff core is supported by observations of surficial features on Bennu [2] and is similarly used in other studies [5, 7, 8, 17]. Simulations are performed with the open-source code LIGGHTS [18] using spherical particles with sliding friction 0.6 and rolling friction 0.2; these values are comparable to those used in other studies, e.g., [5–8], and other parameters used are included in the Supplementary Information. The rotation rate, ω, of the core is chosen so that $\alpha_{eq} = \left(\omega^2 R_{eq}^3\right)/(\gamma M) = 0.7$, where γ defines the strength of gravity and M is the sum of the mass of the core and the mass of debris particles that settle atop the core as described next. In the simulation, as in the analytic model, gravity is centrally directed.

We deposit debris by periodically generating randomly distributed spheres of particles surrounding the core as shown in Fig. 2b. We allow the particles to accelerate to the surface under gravity $\gamma M/R^2$, where as before, R is the distance to the center of the core. We let two complete rotations of the core elapse before the next successive insertion of particles, which we find is ample to allow settling of particles. Settled particles are identified as ones that come to rest and co-rotate with the core. Particles that eject away from the rotating core in the equatorial region are removed from the simulation between successive particle insertions. Such equatorial particle ejections have been reported by others [4–6] and proposed as a source for the formation of satellites which orbit some rotating asteroids. We continue the process of periodic particle insertion until the mass of settled particles equals the mass of the original core (about 8000 particles). As material is deposited, the total rotating mass, M, as well as the equatorial radius R_{eq}, grows, and we fix the rotation rate of the core at its initial value, so by the end of the simulation shown the resulting increase in angular momentum has caused α_{eq} to grow to 0.74.

A completed simulation is shown in the inset to Fig. 2b; simulation details as well as other simulations, e.g. with other masses of settled particles, are included in supplemental materials. Qualitatively, we compare the simulation with Bennu and Ryugu in Fig. 1d by constructing a surface

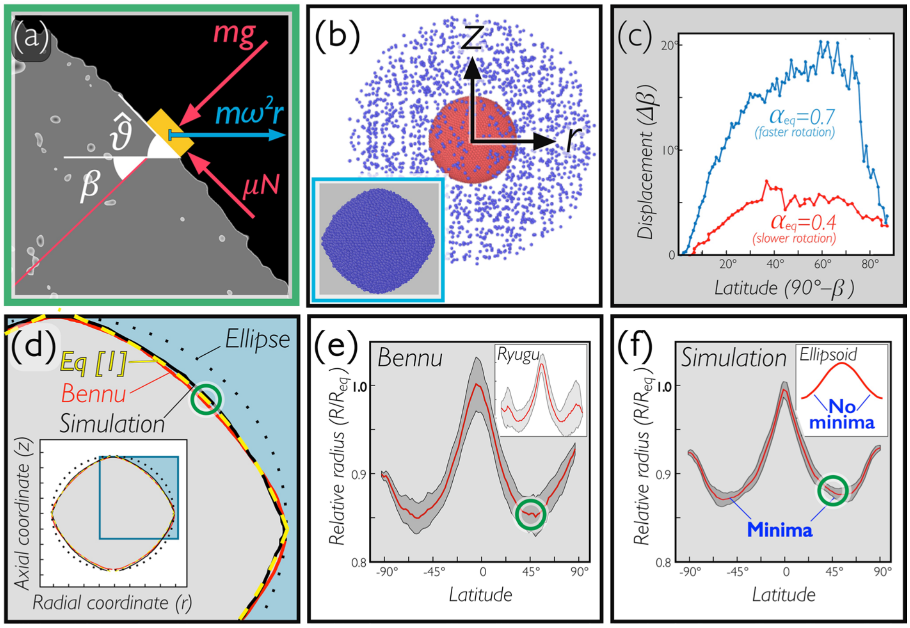

Fig. 2 Model and simulation. **a** Forces acting on mass deposited on surface (enlarged from Fig. 1b). As the mass, m, moves equator-ward, the centrifugal force, $m\omega^2 r$, grows, causing a reduction in normal force, $N = mg\,\sin(\hat{\vartheta} + \beta) - m\omega^2 r\,\sin(\hat{\vartheta})$ and an increase in tangential force, $-\mu N = +m\omega^2 r\,\cos(\hat{\vartheta}) - mg\,\cos(\hat{\vartheta} + \beta)$. Equation 1 gives the resulting angle, $\hat{\vartheta}$, of the surface to the horizontal at which gravity and centrifugal forces are critically balanced. **b** Main image: rotating rigid core in red surrounded by falling blue particles. Inset: settled particles after mass equal to core has been deposited (also plotted in Fig. 1d). **c** Angular displacement of falling particles vs. latitude, showing that particles near pole (0° latitude) and equator (90°) settle near where they land, and that mid-latitude transport grows with rotation rate. **d** Comparison between Eq. (1) using $\alpha_{eq} = 0.99$ and azimuthally averaged simulation and Bennu (for Ryugu, see Supplementary Information). **e** Radii of asteroid surfaces, showing maxima at poles and equator and minima at mid-latitudes; red curves indicate mean radii; grey shows 2σ about mean. **f** Mid-latitude minima also appear in simulation. Inset shows that an ellipse (dotted in (**d**)) has no such minima. Green circle in **d**–**f** identifies mid-latitude point

mesh [19] defined by the centers of the outermost simulated settled particles.

Quantitatively, we compare surface shapes in several ways. First, we test the underlying assumption that decreasing centrifugal force near the poles causes deposited material to accumulate there, while increasing centrifugal forces at lower latitudes causes flow toward the equator. For this purpose, in Fig. 2c we average the changes in latitude from initial to final position of all deposited particles at moderate ($\alpha_{eq} = 0.4$) and high ($\alpha_{eq} = 0.7$) rotational speeds. As expected, deposited particles travel much further at higher angular rotation speed (3 times as far for the speeds shown), and at any rotation speed particles wander furthest from the mid-latitudes, and remain near the poles and equator. This is consistent with the proposition that deposited material accumulates at both poles and equator.

A second comparison involves directly examining the surface profiles, which we do in Fig. 2d by averaging the surface shape around the azimuth for Bennu and for the simulation. Ryugu is considerably more irregular in shape, and we discuss its profile in Supplementary Information. In Fig. 2d, we also include the predicted shape from Eq. 1, which we contrast with an ellipsoidal profile with the same aspect ratio (~ 0.9).

We note that surface shapes of planetary bodies, including diamond-shaped asteroids, are often assessed by their pole-to-equator aspect ratio [5, 6, 10, 11]. Such a scalar measure suffices for ellipsoidal bodies, however as shown in Fig. 2d, diamond-shaped profiles are not ellipsoidal. Therefore we make a third comparison by plotting the azimuthally averaged radius, R, as a function of latitude for asteroids and simulation. This is shown in Fig. 2e, f, where we normalize to the equator by plotting R/R_{eq}. The extent to which the shapes shown differ from ellipsoids can be seen by examination of the region at 45°N highlighted by the green circle in Fig. 2d. This same region is encircled in Fig's 2(e)-(f), where flattening at mid-latitudes produces minima of R/R_{eq}. An ellipsoid would monotonically decrease in radius with latitude, producing no minima. This is shown in the inset to Fig. 2f for an ellipsoid with the same pole-to-equator ratio (0.9). This comparison illustrates the shortcoming on relying on the pole-to-equator ratio for assessing surface shapes.

We emphasize that the problem of spinning asteroids subject to granular relaxation has been intensively

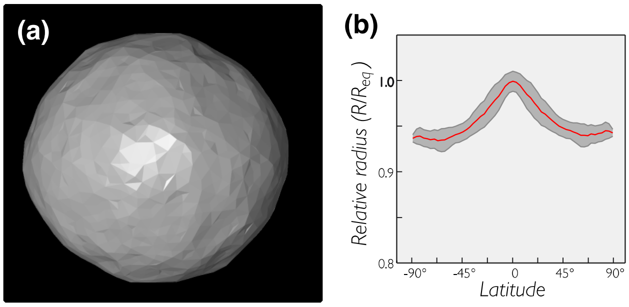

Fig. 3 Effect of reduced centrifugal forcing, using $\alpha_{eq} = 0.4$. **a** Exterior surface of simulation as in Fig. 1d. **b** Predicted radii of asteroid surfaces as in Fig. 2f

analyzed before, notably in Ref. [20]. Crucially, we find that the shape obtained relies on the process of deposition. Just as freshly poured gravel develops a peak at a critical angle that subsides and rounds when agitated, so the poles subside if downslope material is centrifuged sufficiently strongly. By adding new grains, the elevated peak is restored–an effect that is not seen in simulations lacking deposition [5–7].

The essential balance that we have proposed is that adding material by deposition competes against removal of material by centrifugation. Existing literature amply demonstrates that for small, or zero, depositional rate, centrifugation drives material from poles to equator [3–10]. Our Fig. 2 demonstrates that deposition causes poles to elevate–because centrifugation is small there, and deposited particles tend to remain. What remains is to confirm that deposition with slow centrifugation produces an expected simple spheroid. For this purpose, in Fig. 3 we show simulation results for a slowly rotating asteroid ($\alpha_{eq} = 0.4$). All other simulation parameters are identical to the results described previously. In Fig. 3b, we show the relative radius, which indeed displays nearly monotonic decrease in elevation from pole to equator (similar to the inset to Fig. 2f). Comparison with Fig. 3b confirms that the diamond shape, and its accompanying elevation minima at mid-latitudes, only arises due to the combined influence of deposition with centrifugation.

In summary, we have used a simplified model to predict that material deposited uniformly onto a rotating body will tend to accumulate near both the poles and the equator due to a reduction in centrifugal force with latitude. We have confirmed this prediction in simulations, and we have found that both prediction and simulation agree with recent profiles of diamond-shaped asteroids. The model relies on the principle of a critical angle of repose assumed by falling grains deposited on spinning, rubble-pile asteroids. Since the model is driven by deposition, our results suggest first that Bennu and Ryugu acquired their characteristic shapes at early times and second that subsequent reshaping by other effects, such as YORP-induced regolith migration, may be minimal. Our results are in accord with recent analyses that also suggest an early-time formation of the characteristic asteroid shape based on the size and distribution of craters on Bennu [9, 10].

Supplementary Information The online version of this article (https://doi.org/10.1007/s10035-021-01152-z).

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

References

1. Watanabe, S., Hirabayashi, M., Hirata, N., Hirata, N., Noguchi, R., Shimaki, Y., Yabuta, H.: Hayabusa2 arrives at the carbonaceous asteroid 162173 Ryugu—A spinning top–shaped rubble pile. Science **364**(2019), 268–272 (2019)
2. Barnouin, O.S., Daly, M.G., Palmer, E.E., et al.: Shape of (101955) Bennu indicative of a rubble pile with internal stiffness. Nat. Geosci. **12**, 247–252 (2019)
3. Walsh, K.J.: Rubble pile asteroids. Ann. Rev. Astron. Astrophys. **56**, 593–624 (2018)
4. Walsh, K.J., Richardson, D.C., Michel, P.: Rotational breakup as the origin of small binary asteroids. Nature **454**, 188–191 (2008)
5. Walsh, K.J., Richardson, D.C., Michel, P.: Spin-up of rubble-pile asteroids: Disruption, satellite formation, and equilibrium shapes. Icarus **220**, 514–529 (2012)
6. Sugiura, K., Kobayashi, H., Watanabe, S.I., Genda, H., Hyodo, R., Inutsuka, S.I.: SPH simulations for shape deformation of rubble-pile asteroids through spinup: the challenge for making top-shaped asteroids Ryugu and Bennu. Icarus **365**, 114505 (2021)
7. Sánchez, P.: Asteroid evolution: role of geotechnical properties. Proc. Int. Astron. Union **10**, 111–121 (2015)
8. Hirabayashi, M.: Sánchez, DP & Scheeres, DJ "Internal structure of asteroids having surface shedding due to rotational instability." Astrophys J **808**, 63–74 (2015)
9. Walsh, K.J., et al.: Craters, boulders and regolith of (101955) Bennu indicative of an old and dynamic surface. Nat. Geosci. **12**(4), 242–246 (2019)

10. Michel, P., et al.: Collisional formation of top-shaped asteroids and implications for the origins of Ryugu and Bennu. Nat. Commun. **11**(1), 1–11 (2020)
11. Bagatin, A.C., Alemañ, R.A., Benavidez, P.G., Pérez-Molina, M., Richardson, D.C.: Gravitational re-accumulation as the origin of most contact binaries and other small body shapes. Icarus **339**, 113603 (2020)
12. Shinbrot, T., Duong, N.H., Hettenbach, M., Kwan, L.: Coexisting static and flowing regions in a centrifuging granular heap. Granular Matter **9**, 295–307 (2007)
13. Minton, D.A.: The topographic limits of gravitationally bound, rotating sand piles. Icarus **195**, 698–704 (2008)
14. Harris, A.W., Fahnestock, E.G., Pravec, P.: On the shapes and spins of "rubble pile" asteroids. Icarus **199**, 310–318 (2009)
15. Zhang, Y., Zeng, X., Circi, C., Vulpetti, G.: The motion of surface particles for the asteroid 101955 Bennu. Acta Astronaut. **163**, 3–10 (2019)
16. Matsumura, S., Richardson, D.C., Michel, P.: Schwartz, SR & Ballouz, RL "The Brazil nut effect and its application to asteroids." Mon. Not. R. Astron. Soc. **443**, 3368–3380 (2014)
17. Sánchez, D.P., Scheeres, D.J.: Rotational evolution of self-gravitating aggregates with cores of variable strength. Planetary Sp. Sci. **157**, 39–47 (2018)
18. Kloss, C., Goniva, C., Hager, A., Amberger, S., Pirker, S.: Models, algorithms and validation for opensource DEM and CFD–DEM. Prog. Comput. Fluid Dyn., Int. J. **12**, 140–152 (2012)
19. Stukowski, A.: Visualization and analysis of atomistic simulation data with OVITO–the Open Visualization Tool. Modell. Simul. Mater. Sci. Eng. **18**, 015012 (2009)
20. Scheeres, D.J.: Landslides and mass shedding on spinning spheroidal asteroids. Icarus **247**, 1–17 (2015)