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Measurements of the complex permittivity of liquid helium-4 in the millimeter-wave range by a whispering-gallery-mode resonator

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Abstract We report experimental study of electrical properties of liquid helium-4 in the temperature range 1.2-3 K. The experiment is carried out in the millimeter-wave range using a whispering-gallery-mode dielectric resonator, and the complex permittivity of liquid helium is extracted from the data using the resonant perturbation method. The results for temperature dependence of the dielectric constant are consistent with the previous studies. In addition, we find strong enhancement of the loss tangent around the superfluid transition temperature.

Keywords dielectric constant and loss tangent of liquid helium · millimeter waves · whispering-gallery-mode resonator

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1 Introduction

Properties of superfluid helium have been studied for more than 100 years, but still there are interesting issues which require further investigation. For example, unusual electrical activities in the superfluid helium-4 both in low and high frequency ranges were recently observed [1–3]. Therefore, development of robust experimental methods to study the electrical properties of liquid helium is of certain interest. Here, we present measurements of the complex permittivity $\varepsilon = \varepsilon' - j\varepsilon''$ of liquid helium-4 in the millimeter-wave range at temperatures 1.2-3 K employing a whispering-gallery-mode (WGM) dielectric resonator.

As well known the liquid helium-4 is a monoatomic liquid comprised of weakly interacting atoms. Its polarization results from the elastic deformation of the electron shell and has the characteristic frequencies in the optical range. Usually, well below these frequencies the complex part of the permittivity, which is associated with electron motion dumping, is neglected. Correspondingly, for description of the

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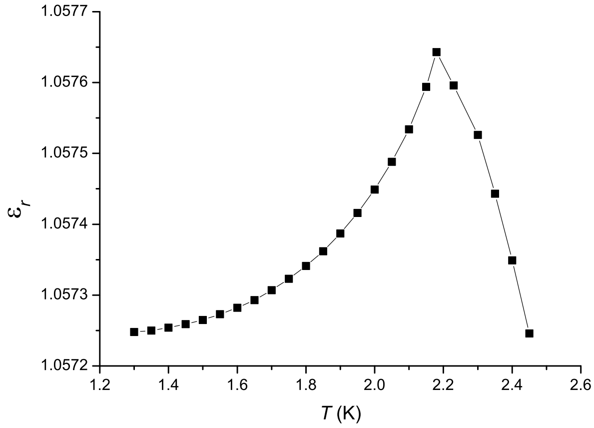

Fig. 1 Temperature dependence of the dielectric constant of liquid helium-4 according to [10].

electrical properties of liquid helium the Clausius-Mossotti equation is commonly used

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{4\pi\alpha\rho}{3M} \tag{1}$$

where $\varepsilon_r = \varepsilon/\varepsilon_0$ is the dielectric constant ($\varepsilon_0 = 8.854 \times 10^{-12}$ F/m is the permittivity of vacuum), ρ is the density of liquid, M is the molecular weight, and α is the polarizability of helium atom. In the above approximation, the polarizability α is assumed to be constant. Under this assumption the temperature dependence of the liquid helium density can be determined [4]. This fact provided a great interests in the accurate measurements of the dielectric constant ε_r by the capacitance method [5–9]. The experimental results were summarized by Donnelly and Barenghi [10] and are plotted in Fig. 1.

The dielectric constant of liquid helium was also measured in the centimeter-wave range by employing a high-quality-factor ($Q \sim 10^6$) superconducting 9 GHz cavity resonator [11]. In this method, the dielectric constant is obtained as

$$\varepsilon_r = \left(\frac{f_e}{f_f}\right)^2, \tag{2}$$

where f_e and f_f is the resonant frequency of empty cavity and cavity completely filled with liquid, respectively. The results were found in reasonable agreement with the conventional capacitance measurements. The detection of the dielectric losses in liquid helium was beyond the method sensitivity, and the loss tangent of liquid helium defined as tan $\delta = \varepsilon''/\varepsilon'$ was concluded to be less than 5×10^{-6}.

In a later work, the electrical properties of liquid helium were studied by resonant techniques using a spherical WGM dielectric resonator immersed into the liquid and operating at the resonant frequency of 36.5 GHz [12]. Around the superfluid transition temperature, the authors observed a significant increase in the dielectric losses of liquid helium which they attributed to the interaction of the microwave electromagnetic field with the superfluid fraction of liquid, but no detailed explanations were given.

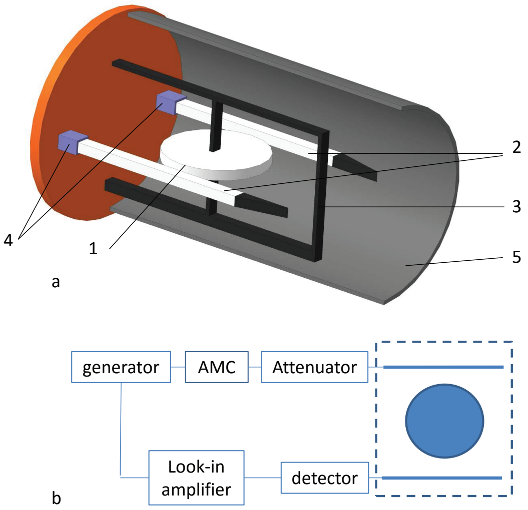

Fig. 2 (color online) Sketch of the experimental setup (a) and block diagram of the electronics (b).

In this work, we employ a WGM resonator in the form of a dielectric disc operated at frequencies 170-180 GHz to detect small changes in the dielectric properties of liquid helium surrounding the resonator. In particular, both the dielectric constant and loss tangent of liquid helium are measured at the temperatures above and below the superfluid transition. Our work demonstrates that WGM resonator provides an excellent experimental tool to study electrical properties of liquid helium.

2 Experimental method

The sketch of the experimental cell is shown in Fig. 2a. The WGM resonator, a quartz disk (1) of radius $R \approx 10$ mm and thickness $d \approx 1$ mm, was mounted on a metal frame (3) inside the cell (5) by means of two spring-loaded contacts in the center. The plane of the disk was placed horizontally. The traveling waves propagating in the hollow metal transmission waveguides (4) could be coupled to the resonator by means of two tapered dielectric waveguides (2) inserted in the metal waveguides and placed in close proximity to the disk (see Fig. 2a). The distance between the disk and coupling waveguides could be adjusted. The tapered end of the coupling waveguide was covered with absorbing material to avoid reflection of the microwave signal from the waveguide end. The orientation of this system was chosen such as to excite resonant WGE$_{m,n}$ (quasi-TE) modes with microwave electric field having predominantly the radial component [13]. Here, the subscripts m and n denote the order of azimuthal and radial dependence, respectively. As is well known, the radial distribution of WGM inside the disk is described by the Bessel functions $J_m(kr)$ with the argument kr of the order of m, where m is a large

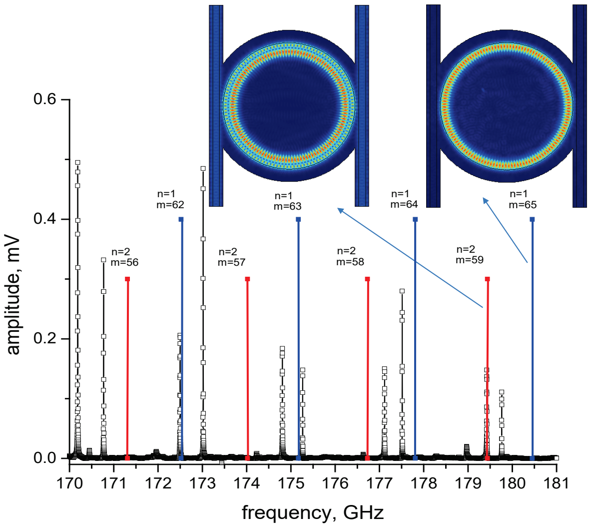

Fig. 3 (color online) Transmission spectrum of the empty cell (○) measured at $T = 2$ K, 🟥 and 🟦 - calculated resonant frequencies. Inset: the electric field distribution of $\mathrm{WGE}_{59,2}$ (left) and $\mathrm{WGE}_{65,1}$ (right) resonant modes obtained using FEM calculations (COMSOL).

number [14]. In this case, the electromagnetic energy is predominantly confined between the cylindrical boundary R and a slightly smaller radius, while it exponentially decays everywhere else. To find exact resonant frequencies of WGMs, the electromagnetic field distribution has to be calculated numerically using general expressions through the cylindrical functions of complex order [15] or using the finite-element method (FEM) calculation [16].

To excite oscillations in the millimeter-wave range, we used a 2-20 GHz signal synthesizer followed by an amplifier-multiplier chain (see Fig. 2b). The microwave power delivered into the transmission waveguide coupled to the cell could be changed in the range from a few microwatt to about 1 milliwatt by an attenuator. The second transmission line, where the traveling wave was excited by the WGMs in the resonator, was connected to a room temperature power detector having sensitivity of about 500 V/W. The transmission spectrum of the WGM resonator was measured by pulse-modulating the input microwave power at 1 kHz and measuring the demodulated response of the detector using a lock-in amplifier. In the experiment, the cell was completely filled with the liquid helium, and the frequency shift and the quality factor variation due to the presence of liquid helium was measured. From this data, the dielectric constant and loss tangent of liquid helium could be determined using the resonant perturbation method.

3 Results

The transmission spectrum of the empty cell measured at $T = 2$ K in the frequency range 170-180 GHz is shown in Fig. 3. In the given range, the observed spectrum consists of the roughly equally-spaced pairs of resonant modes. To identify the

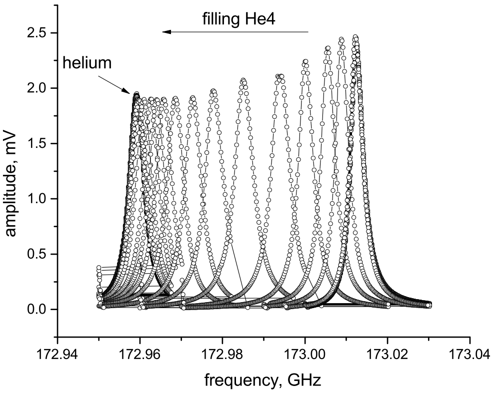

Fig. 4 Variation of the transmission spectrum as the cell is filled with the liquid helium-4 at $T = 4.2\ K$.

modes in each pair, we carried out FEM calculations of the electromagnetic field distribution using COMSOL. It is found that each pair presumably corresponds to $n = 1$ and $n = 2$ modes of different azimuthal order, which have the highest quality factor. An example of field distributions for $\mathrm{WGE}_{m,1}$ and $\mathrm{WGE}_{m',2}$ adjacent modes is shown in the inset of Fig. 3. Some discrepancies between exact positions of the experimentally observed and calculated modes can be presumably attributed to the deviation of the actual values for disk radius and dielectric constant of the disk material from those used in the calculations. We note that a similar double-peak structure was observed in the transmission spectrum of a microdisk WGM resonator [17].

The evolution of one of the resonant modes ($\mathrm{WGE}_{62,1}$) as the cell is filled with liquid helium is shown in Fig. 4. Strong variations in the resonance frequency and quality factor, the latter being observed as the change in the transmission intensity, confirms that the WGM resonator is sensitive to the presence of the surrounding liquid.

Measurements of the temperature dependence of the permittivity of liquid helium were done using several modes, and all measurements showed similar results. In the experiment, the temperature of the cell T was varied in the range 1.2-3 K, and the transmission spectrum similar to the one shown in Fig. 4 was recorded for different values of T. The complex permittivity of liquid helium for each temperature was extracted from the change of the resonant frequency and the quality factor of the given resonant WGM with respect to those for the empty cell using the resonance perturbation method [19,18]. In general, the change in the complex angular frequency of WGM is given by

$$\frac{\omega_2 - \omega_1}{\omega_2} = -\left(\frac{\varepsilon/\varepsilon_0 - 1}{2}\right)\frac{\iiint\limits_{Vs} \mathbf{E}_1\mathbf{E}_2\, dV}{\iiint\limits_{Vc} |\mathbf{E}_1|^2\, dV}, \tag{3}$$

where ω_1 and ω_2 are the angular frequencies for empty and filled cell, respectively, $\mathbf{E}_1$ and $\mathbf{E}_2$ is the microwave electric field for empty and filled cell, respectively,

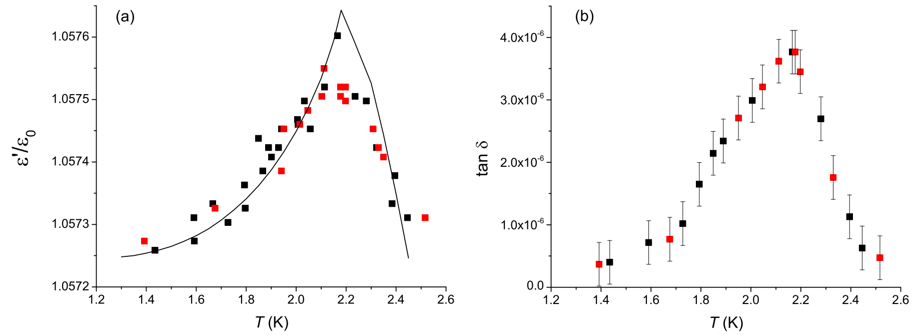

Fig. 5 (color online) The dielectric constant $\varepsilon'/\varepsilon_0$ (a) and the loss tangent $\tan\delta = \varepsilon''/\varepsilon'$ (b) of liquid helium as a function of temperature. The data plotted by ■ and 🟥 are for increasing and decreasing temperature, respectively. The solid line shows ε_r from Ref. [10], see Fig. 1.

and the integrals are over the volumes of the liquid sample Vs and dielectric disk Vc, respectively. The resonant frequency ω_r and the quality factor Q of the WGM can be determined from

$$\omega = \omega_r + j\omega_i, \quad \omega_r = 2\pi f, \quad Q = \frac{\omega_r}{2\omega_i}, \tag{4}$$

which allows us to rewrite the left-hand side of Eq. (3) as

$$\frac{\omega_2 - \omega_1}{\omega_2} \approx \left(\frac{f_2 - f_1}{f_2}\right) + j\left(\frac{1}{2Q_2} - \frac{1}{2Q_1}\right), \tag{5}$$

where we assumed that $f_1 \approx f_2$ and $\omega_i << \omega_r$. Thus, from Eqs. (3) and (5) we obtain

$$\begin{aligned} \frac{f_2 - f_1}{f_2} &= \left(\varepsilon'/\varepsilon_0 - 1\right)\frac{C}{2}, \\ \frac{1}{Q_2} - \frac{1}{Q_1} &= \varepsilon''/\varepsilon_0 C. \end{aligned} \tag{6}$$

where

$$C = \frac{\iiint\limits_{Vs} \mathbf{E}_1\mathbf{E}_2\, dV}{\iiint\limits_{Vc} |\mathbf{E}_1|^2\, dV}. \tag{7}$$

In the resonant perturbation method, the parameter C is assumed to be a constant independent of the properties of the liquid. In practice, it is determined directly by a calibration procedure using a liquid with known values of ε' and ε''. Instead, we used FEM calculation to determine the value of C for a given WGM. Using the obtained value, we determined ε' and ε'' from the experimental values of f_i and Q_i, $i = 1, 2$. The results are shown in Fig. 5.

4 Discussion

Our experimental values for the dielectric constant of liquid helium are found to be in good agreement with the previous studies. This is shown by comparison with the data from Ref. [10] given by the solid line in Fig. 5(a). This demonstrates that our experimental method is suitable for study of the electrical properties of liquid helium. In addition, we observed a strong enhancement of the loss tangent of liquid near the superfluid transition point. A similar effect was reported earlier employing a spherical WGM resonator operating at the frequency 36.5 GHz [12]. Surprisingly, we observe almost an order-of-magnitude enhancement, which is much larger comparing with the result in Ref. [12]. While we can not provide any theoretical explanations for such a large enhancement, we note that the microwave frequency range 170-180 GHz used in this experiment is where the unusual electrical activity was observed in the superfluid helium-4. Further experimental studies employing this method are needed in the future.

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